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CONDITIONING DISCRETE FRACTURE NETWORK MODELS OF GROUNDWATER FLOW

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Abstract. Many geological formations consist of crystalline rock that have very low matrix permeability but allow flow through an interconnected network of fractures. Understanding the flow of groundwater through such rocks is important in considering disposal of radioactive waste in underground repositories. A specific area of interest is the conditioning of fracture transmissivities on measured values of pressure in these formations. While there are existing methods to condition transmissivity fields on transmissivity, pressure and flow measurements for a continuous porous medium, considerably less work has been devoted to conditioning discrete fracture networks. This article presents two new methods for conditioning fracture transmissivities on measured pressures in a discrete fracture network. The first approach adopts a linear approximation when fracture transmissivities are mildly heterogeneous, while the minimisation of a suitable objective function is undertaken when fracture transmissivities are highly heterogeneous. The second conditioning algorithm is a Bayesian method that finds a maximum a posteriori (MAP) estimator which maximises the posterior distribution defined by Bayes' theorem using information from the prior distribution of fracture transmissivities and observations in the form of measured pressures. The conditioning methods are tested on two separate, large scale test cases that model a potential site for radioactive waste disposal. Results from these test cases are shown and comparisons between the two conditioning methods are made.

Key Words. Conditioning, Groundwater Flow, Discrete Fracture Network, Finite Element Methods.

1. Introduction

Many geological formations consist of crystalline rock that have very low matrix permeability but allow flow through an interconnected network of fractures. Understanding the flow of groundwater through such rocks is important in considering disposal of radioactive waste in underground repositories. In our work it is assumed that there is no interaction between groundwater flow (and the pollutants it may carry) in the fractures and the surrounding rock matrix; this setting is known as a discrete fracture network (DFN). A DFN is characterised by the properties of the fractures, namely, the density of the fractures, their size, orientation and transmissivity. The transmissivity of a fracture is defined as the rate of groundwater flow per unit pressure gradient. It thus gives a measure of the ease with which groundwater can pass through a material (a fracture in our case). In our work the fractures are modelled such that the fracture walls are represented as two parallel plates with

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groundwater flowing between them. In this setting, the fracture transmissivity is proportional to the cube of the fracture aperture (width between the fracture walls) [21], and both the aperture and transmissivity are constant over the fracture. In a problem of practical relevance, there are generally too many fractures for all of their properties to be measured. To remedy this problem when numerically modelling a DFN, a stochastic approach can be exploited; here distributions of fracture properties (aperture, length, orientation, location) are inferred from field measurements and are subject to uncertainty [12, 17]. Realisations of fractures can be generated in a given domain with fracture properties (aperture, length, orientation, location) sampled from distributions consistent with observed measurements. Fractures with known properties can also be included deterministically in a model of this type. This paper develops two numerical methods in a DFN setting to condition fracture transmissivities on measured values of the pressure available from test site data. The groundwater flow equation [1] can be used to calculate the pressure in a fracture. DFNs are modelled numerically with suitable boundary conditions at both fracture intersections and the domain boundaries [8]. Our work exploits a finite element approach to modelling groundwater flow in a DFN. Alternative numerical methods for solving flow in a DFN are discussed in Jing [10].

The problem considered in this article can be summarised in a continuous setting (before discretisation of the domain) as follows: determine $\mathbf{T}$ such that

$$\|P(\mathbf{X}_M) - \mathbf{P}_M\| = \min!,$$

under the constraint,

$$\nabla \cdot (\mathbf{T}\nabla P) = 0 \text{ in } \Omega,$$

subject to appropriate boundary conditions. Here, $\mathbf{T}$ is a vector of fracture transmissivities (containing hundreds or thousands of fracture transmissivities), $\mathbf{P}_M$, $M \geq 1$, is a vector of measured pressures that are to be matched, $\mathbf{X}_M$, $M \geq 1$, denotes the locations of the measurement points, P is the pressure, Ω is the domain of the fracture network and $\|\cdot\|$ denotes an appropriate norm. Boundary conditions are imposed both on the boundaries of the problem domain Ω, as well as at fracture intersections. Generally, there are far less pressure measurements than fracture transmissivities.

When studying DFNs it is common to use more than one realisation of the DFN due to uncertainties in the fracture properties. In this setting, the geometry of the fractures is sampled from various distributions; thus each realisation will have different fracture geometry. Calibration is the process of modifying input parameters to a model until the output from the model matches observed data. Each realisation should be calibrated using as much available data as possible. In our work the model parameters are the fracture transmissivities and they are conditioned on measured pressures.

While there are existing methods to condition transmissivity fields on transmissivity, pressure and flow measurements for a continuous porous medium [9, 19, 20], there is considerably less work within the literature on conditioning DFNs. An exception is the recent work by Frampton and Cvetkovic [5] who condition the parameters of a fracture transmissivity distribution in a DFN setting, but they do not condition fracture transmissivities directly. Conditioning fracture transmissivities on pressure or flow values is a complex problem because the measured pressures are dependent on all the fracture transmissivities in the DFN.

In this article, we present two new methods for conditioning fracture transmissivities in a DFN on measured pressure values. Both methods consider one realisation,