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STABILITY CRITERIA AND MULTIPLE BIFURCATION ANALYSIS FOR SOME NONLINEAR CONTINUOUS-TIME COUPLED SYSTEMS WITH MULTIPLE DELAYS

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Abstract. A coupled system, which consists of multiple delayed neural network loops, is proposed and a detailed analysis of the asymptotic behavior of the zero solution is included. The stable regions and all possible bifurcations, which depend on multiple parameters, are given in a geometrical way for several specific cases.

Key Words. stability, multiple bifurcations, neural network loops, delay.

1. Introduction

Consider a continuous-time Hopfield neural network with m identical subsystems which have n nonidentical neurons with n delays and no self-connection of the following form:

(1.1)

$$
\begin{array}{ll}
subsystem\ 1 & \left\{\begin{array}{rcl}
x'_{11}(t) &=& -a_1x_{11}(t)+\beta_nf_n(x_{1n}(t-\tau_n))+\epsilon_1g_m(x_{mn}(t-\tau_n))\\
x'_{12}(t) &=& -a_2x_{12}(t)+\beta_1f_1(x_{11}(t-\tau_1))\\
&\vdots&\\
x'_{1n}(t) &=& -a_nx_{1n}(t)+\beta_{n-1}f_{n-1}(x_{1(n-1)}(t-\tau_{n-1}))
\end{array}\right.\\
subsystem\ 2 & \left\{\begin{array}{rcl}
x'_{21}(t) &=& -a_1x_{21}(t)+\beta_nf_n(x_{2n}(t-\tau_n))+\epsilon_2g_1(x_{1n}(t-\tau_n))\\
x'_{22}(t) &=& -a_2x_{22}(t)+\beta_1f_1(x_{21}(t-\tau_1))\\
&\vdots&\\
x'_{2n}(t) &=& -a_nx_{2n}(t)+\beta_{n-1}f_{n-1}(x_{2(n-1)}(t-\tau_{n-1}))
\end{array}\right.\\
\vdots & \\
subsystem\ m & \left\{\begin{array}{rcl}
x'_{m1}(t) &=& -a_1x_{m1}(t)+\beta_nf_n(x_{mn}(t-\tau_n))\\
&& \qquad +\epsilon_mg_{m-1}(x_{n(m-1)}(t-\tau_n))\\
x'_{m2}(t) &=& -a_2x_{m2}(t)+\beta_1f_1(x_{m1}(t-\tau_1))\\
&\vdots&\\
x'_{mn}(t) &=& -a_nx_{mn}(t)+\beta_{n-1}f_{n-1}(x_{m(n-1)}(t-\tau_{n-1}))
\end{array}\right.
\end{array}
$$

where x_{ij} denotes the activation of the jth neuron within the ith subsystem, a_j is the internal decay of the neurons, β_j and ϵ_i are the connection weights, nonnegative integers τ_j denotes the synaptic transmission delays, which corresponds to the time when a signal is emitted by the $(j-1)-$th neuron, and becomes available for the

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$j-$th neuron. For above notations, $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$. $f : \mathbb{R} \to \mathbb{R}$ is the activation function.

In the past twenty years, there have been an increasing interest on the study of the dynamical evolution of nonlinear delayed coupled systems. The attractiveness of nonlinear coupled systems may lie in their possible modeling the interaction dynamics among neurons (such as Hopfield/Cohen-Grossberg neuron networks [7, 14, 15, 16]) or oligopolists (such as Cournot duopoly models [20]) etc. Among the most widely studied phenomena is synchronization, where individual networks oscillate at the same frequency and phase when coupled. According to the learning rules of Hebb [13]: synchronous activation increases the synaptic strength, whereas asynchronous activation decreases the synaptic strength.

It is known that the delay increases the dimensionality, and hence the complexity. It is natural for the inclusion of time delay in the realistic consideration of finite transmission of the interaction, such as the propagation of information through a network node or "synapse". Now great efforts have been made on those domains where delay is not the major factor, or where there occur rich dynamics.

For the study of existence and stability of periodic solutions with spatiotemporal symmetries in delay-coupled neural networks of delay-differential equations, we refer the reader to Refs. [5, 9, 11, 12, 17, 23, 26], where multiple periodic/steady-state solutions can be obtained and observed by equivariant Hopf/fold bifurcations from the trivial zero equilibrium solution. But each of the neurons of the networks is described by a one-dimensional nonlinear differential equation systems.

For the study of existence and stability of periodic solutions in delay-coupled asymmetric neural networks, we refer the reader to Refs. [1, 25], where Hopf/fold bifurcations were discussed and the mechanism of how delay affects neural dynamics and learning is explored[1]. But each of the neurons of the networks is also described by a one-dimensional nonlinear differential equation systems.

Moreover, there is an increasing interest in some nonlinear delayed neural networks coupled by two sub-networks [3, 19, 24]. In [3], the authors discussed the stability and bifurcations in the delayed neural network coupled by a pair of three-neuron sub-networks without internal delays. But they did not deal with the direction and stability of Hopf bifurcation and the possible spatio-temporal patterns of bifurcating periodic oscillations. In [19, 24], a neural network coupled by a pair of two-neuron sub-networks is investigated, which contains the time delay not only in the coupling but also in the internal connection. Yet one can find that all the delays have the same size in [24].

Motivated by proposing a more generalized model than those in [3, 24], we consider model (1.1), which consists of multiple nonlinear delayed neural network loops by delay coupling.

It is well-known that an artificial neural network (ANN) is composed of many artificial neurons that are linked together according to a specific network architecture. The objective of the neural network is to transform the inputs into meaningful outputs. Artificial neural networks are inspired by the learning processes that take place in biological systems, which try to imitate the working mechanisms of their biological counterparts. Since McCulloch and Pitts's first formal model of the elementary computation neuron in 1943 [22], which could perform arithmetical logic operations, a great amount of ANN models have been proposed and developed according to the purposes of the applications or theoretical analysis. The applications of ANNs range from classification (including pattern recognition, feature extraction, detection and clustering, image matching), noise reduction (recognizing patterns in the inputs and produce noiseless outputs), prediction (extrapolation