INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 9, Number 1, Pages 115–131

SOME MULTISCALE RESULTS USING LIMITED GLOBAL INFORMATION FOR TWO-PHASE FLOW SIMULATIONS

LIJIAN JIANG, JØRG E. AARNES, AND YALCHIN EFENDIEV

Abstract. In this paper, we present the analysis of recently introduced multiscale finite element methods that employ limited global information. In particular, these methods use single-phase flow information for the construction of more accurate solution for two-phase immiscible flow dynamics in heterogeneous porous media. We consider the analysis of Galerkin multiscale finite element method as well as mixed multiscale finite element method. Our analysis assumes that the fine-scale features of two-phase flow dynamics strongly depend on single-phase flow. Under this assumption, we present the analysis of multiscale finite element methods that use single-phase flow information. Numerical results are presented which demonstrate that MsFEM using limited global information is more accurate and converges as the coarse mesh size decreases.

Key Words. Galerkin multiscale finite element method, mixed multiscale finite element method, global information, two-phase flows

1. Introduction

Subsurface flows are often affected by heterogeneities in a wide range of length scales. It is therefore difficult to resolve numerically all of the scales that impact transport. Typically, upscaled or multiscale models are employed for such systems. The main idea of upscaling techniques is to form coarse-scale equations with a prescribed analytical form that may differ from the underlying fine-scale equations. In multiscale methods, the fine-scale information is carried throughout the simulation and the coarse-scale equations are generally not expressed analytically, but rather formed and solved numerically.

Our purpose in this paper is to analyze multiscale finite element methods for two-phase immiscible flows that employ single-phase flow information. A multiscale finite element method was first introduced in [19] and takes its origin from the pioneering work [9]. Its main idea is to incorporate the small-scale information into finite element basis functions and capture their effect on the large scale via finite element computations. The multiscale method in [19] shares some similarities with a number of multiscale numerical methods, such as residual free bubbles [10, 26], variational multiscale method [20], two-scale finite element methods [25], two-scale conservative subgrid approaches [4]. We remark that special basis functions in finite element methods have been used earlier in [9, 7]. The multiscale finite element methodology has been modified and successfully applied to two-phase flow simulations in [21, 13, 1] and extended to nonlinear partial differential equations [18, 17].

Received by the editors October 1, 2008 and, in revised form, March 17, 2009.
2000 *Mathematics Subject Classification.* 65N30, 34E13, 34K28.

Recently, a number of upscaling and multiscale approaches that employ single-phase flow information in upscaling two-phase flow problems have been introduced. For example, in [12], the authors propose adaptive local-global technique that employs the solution of single-phase flow problem for computation of accurate upscaled properties. In [1] and later in [16], the single-phase flow solution is used to construct multiscale basis functions for accurate simulations of two-phase flow equations. The goal of this paper is to provide analysis of two multiscale finite element methods that use single-phase flow information to construct basis functions. These multiscale techniques have advantages if the fine-scale features of two-phase flow dynamics strongly depend on single-phase flow. In particular, we assume that two-phase flow pressure dynamics strongly depends on single-phase flow pressure. This assumption is shown for channelized permeability in [16]. The convergence rate of multiscale finite element method is obtained under this assumption. We analyze both Galerkin multiscale finite element method as well as mixed multiscale finite element method.

In the paper, we present numerical results which demonstrate that MsFEM using limited global information is more accurate compared to MsFEM which only uses local information to construct basis functions. Moreover, MsFEM with limited global information converges as the coarse mesh size decreases. In our numerical results, the permeability fields from SPE Comparative Solution Project [15] (also known as SPE 10) is used. These permeability fields have channelized structure and a large contrast. Because of channelized structure of the permeability fields, the localized approaches do not perform well. Our numerical results show that one can achieve high accuracy if MsFEM with limited global information is used.

The paper is organized as follows. In the next section, we present a brief discussion of two-phase flow and single-phase flow equations and a brief description of main results. Section 3 is devoted to the analysis of Galerkin multiscale finite element method. In Section 4, we present analysis for mixed multiscale finite element method. In section 5, we present numerical results.

2. Motivation

In this section, we briefly present single-phase and two-phase flow equations neglecting the effects of gravity, compressibility, capillary pressure and dispersion on the fine scale. Porosity, defined as the volume fraction of the void space, will be taken to be constant and therefore serves only to rescale time. The two phases will be referred to as water and oil and designated by the subscripts w and o, respectively. We can then write Darcy's law, with all quantities dimensionless, for each phase j as follows:

$$\mathbf{v}_j = -\lambda_j(S)k\nabla p, \tag{1}$$

where $\mathbf{v}_j$ is phase velocity ($j = w, o$), S is water saturation (volume fraction), p is pressure, $\lambda_j = k_{rj}(S)/\mu_j$ is phase mobility, where k_{rj} and μ_j are the relative permeability and viscosity of phase j respectively, and k is the permeability tensor, which is here taken to be diagonal.

Combining Darcy's law with conservation of mass, $div(\mathbf{v}_w + \mathbf{v}_o)$=0, allows us to write the flow equation in the following form

$$div(\lambda(S)k\nabla p) = f, \tag{2}$$

where the total mobility $\lambda(S)$ is given by $\lambda(S) = \lambda_w(S) + \lambda_o(S)$ and f is the source term which represents wells. The saturation dynamics affects the flow equations.