

DECOUPLED AND MULTIPHYSICS MODELS FOR NON-ISOTHERMAL COMPOSITIONAL TWO-PHASE FLOW IN POROUS MEDIA

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Abstract. A new multiphysics model for two-phase compositional flow is presented. It is designed to fit the level of model complexity to that of the flow and transport processes taking place in a given region of the domain. Thus, the model domain is divided into a subdomain which accounts for two-phase compositional processes and another in which single-phase transport is described. A coupling of the simple and complex equations gives rise to an efficient model. Special interest is placed in the discretization of the two-phase compositional model in a finite-volume context and an IMPES time scheme with decoupled pressure and transport equations. For optimal subdomain determination, an easy-to-handle, adaptive scheme is presented. The practical usability is demonstrated on a real live problem from carbon dioxide sequestration.

Key Words. multiphysics, domain decomposition, multiphase flow, compositional, sequential, decoupled formulation

1. Introduction

The modeling of flow processes in porous media is used in many environmental and engineering applications. To gain insight into increasingly complex processes, models in use become increasingly complex which has obvious consequences for the computational effort. When considering large systems, detailed measurements (such as saturations of contaminants or concentrations of certain components) are usually only available in relatively small regions in which complex processes are assumed to occur, whereas fewer and simpler measurements may be available in other parts of the considered domain. This poor spatial resolution of measurement data in large parts of the model domain questions the use of highly complex and detailed models. More important, complex processes often occur only in a small part of the domain of interest. In order to account for these factors, the coupling of a two-phase compositional model to a single-phase transport model is presented. This allows for the use of the complex two-phase compositional model only in the parts of the model domain where complex processes occur, whereas other parts of the domain are resolved by a simpler single-phase model.

Coupling of different models for the simulation of complex processes was for example originally discussed in the context of domain decomposition methods (e.g., [17]). This was the motivation for [14], where the authors develop a multiblock framework to couple fully implicit and sequential two-phase flow models. Further developments resulted in multiphysics coupling of single-phase (implicit or sequential), two-phase (implicit or sequential) and black-oil (implicit only) models as

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published in [15] and [16]. However, the coupling of the different models involved a nonlinear iterative solver to match the coupling conditions at the interfaces. Furthermore, the black-oil model was only presented in a fully implicit formulation which in turn involves nonlinear solvers. Due to the relatively simple form of the pressure equations, the resulting smaller matrices and the noniterative solution scheme, sequential models require less computational effort per timestep, making them attractive in wide areas of application ([18]). The black-oil model also contains rather strong assumptions (water phase contains no other components than water, gas phase is only made up of light hydrocarbons), limiting its applicability to certain areas.

To account for the mentioned data collection problem, a multi-scale multi-physics model approach was presented in [7]. In this study, the velocity is assumed to be divergence-free. This assumption, however, is not true for compositional multiphase flow, since the mixing of phases influences the total fluid volume.

We introduce a sequential multiphysics model for two-phase compositional flow without the need for nonlinear solvers and without restrictions concerning miscibility. For this purpose, we incorporate the decoupled formulation for two-phase compositional flow introduced and analyzed in [3] and expanded to other formulations of the pressure equation in [6]. We want to concentrate on applications like enhanced recovery of contaminated soils or carbon dioxide sequestration, where we assume strongly advection-dominated problems and hence neglect diffusion and capillarity.

2. Physical Model

2.1. General balance equation. The mass balance equation per component in a multiphase compositional model neglecting diffusion is (e.g., [1])

$$(1) \quad \frac{\partial C^\kappa}{\partial t} = -\nabla \cdot \sum_{\alpha} \mathbf{v}_{\alpha} \varrho_{\alpha} X_{\alpha}^{\kappa} + q^{\kappa},$$

where $C^{\kappa} = \phi \sum_{\alpha} S_{\alpha} \varrho_{\alpha} X_{\alpha}^{\kappa}$ denotes the total concentration of component κ , and ϕ is the porosity. Moreover, S_{α} and ϱ_{α} are the saturation and the density of phase α , respectively, while X_{α}^{κ} is the mass fraction of component κ in phase α . The source term of component κ in the unit mass per time is denoted by q^{κ} . Given the mobility $\lambda_{\alpha} = \frac{k_{r\alpha}}{\mu_{\alpha}}$ – where μ_{α} and $k_{r\alpha}$ are the dynamic viscosity and the relative permeability of phase α , respectively – , the pressure p , the permeability tensor \mathbf{K} and the gravity vector \mathbf{g} , the velocity of phase α is given by the extended Darcy law as

$$(2) \quad \mathbf{v}_{\alpha} = \lambda_{\alpha} \mathbf{K} (-\nabla p + \varrho_{\alpha} \mathbf{g}).$$

2.2. Pressure equation. As a physical constraint, the pore space of the porous medium must always be filled with a mixture of fluids. That is, the sum over the volumes of the phases inside a control volume must equal the pore volume, or

$$(3) \quad \left(\sum_{\kappa} C^{\kappa} \right) \cdot \left(\sum_{\alpha} \frac{\nu_{\alpha}}{\varrho_{\alpha}} \right) - \phi = 0,$$

where $\nu_{\alpha} = \frac{S_{\alpha} \varrho_{\alpha}}{\sum_{\alpha} S_{\alpha} \varrho_{\alpha}}$ denotes the mass fraction of phase α . Due to compressibility and mixing effects, the volume of a mixture is affected by changes in pressure or total concentration. These volume changes must be taken into account in the