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COMPRESSIBLE LATTICE BOLTZMANN METHOD AND APPLICATIONS

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Abstract. Lattice Boltzmann Method (LBM) is a novel numerical method for flows simulations. Compared with classic methods of Finite Difference Method, Finite Volume Method and Finite Element Method, LBM has numerous advantages, including inherent parallelization and simplicity of boundary condition treatment. The LBM usually has a constraint of incompressible fluid (Mach number less than 0.4). A variant of the LBM is studied and used to deal with compressible fluid with Mach number up to 0.9 in this paper. Special emphasis is placed on mesh generation of 3-D complete geometry in Cartesian coordinate system. Numerical experiments are fulfilled in 2-D and 3-D compressible flows. Performance evaluation of the algorithm demonstrates high parallel efficiency and prefect scalability. Numerical results indicate that the LBM is successful with the simulation of compressible fluid.

Key words. Lattice Boltzmann Method, Compressible fluid, Cartesian Mesh generation.

1. Introduction

There are two different ways to numerically simulate fluid flow, i.e., one based on macro-continuous model from top to button and one of micro-discrete model from button to top. From Euler equations and Navier-Stokes equations, classical numerical methods of finite different method, finite volume method and finite element discretize the equations, obtain the liner systems and solve the systems. While such from top to button approaches intuitive, there are still many deficiencies. For example, in such method, often focus on analysis from the continuous differential equations to the discrete algebraic equations of the truncation error, but ignored the discrete process of conservation of certain physical quantities. Furthermore, in dealing with complex flow systems, to solve this type of nonlinear differential equations is very difficult or impossible.

In recent years, much attention of the Lattice Boltzmann method[1, 2] (hereinafter referred to as LBM) belong to micro-discrete model based on from button to top approach. In the LBM method, the fluid is an abstract for a large number of micro-particles, and these micro-particles in a discrete lattice migrate and collide in accordance with a simple movement rules. By the statistical particle can get the macro movement of fluid characteristics. This particle properties of LBM method also make the conventional numerical methods do not have many unique advantages, such as the physical image is clear, the boundaries treat easier and the computing is nature of parallel processing and so on. LBM method also provides the possibility and the reality of macro and micro. It is both a direct calculation of

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the viscous fluid, and also approximate Navier-Stokes equations under certain conditions. At the same time, LBM modeling is achieved with a simple mathematical modeling of complex systems, it also breaks the traditional concept of modeling for other complex systems and provides a new way. The evolution process of LBM method (in particular, LBGK model) is very simple and clear, its program more concise. Lattice Boltzmann method involved in the calculations are localized, with the natural parallelism, very suitable for large-scale parallel computer. Because of these advantages, LBM method is considered a promising method of calculation and raised a strong interest.

Currently, LBM has been in the multi-phase flow, porous media flow, particulate flow, reacting flow, magnetic fluid mechanics and bio-mechanics have achieved great success, their efficiency, accuracy and robustness have been widely confirmed. The traditional LBM methods can only handle low-speed incompressible fluid flow, can not be used for high-speed moving objects such as aircraft simulation, thereby limiting the application of the method. To calculate the high-speed compressible fluid flow, researchers have begun a new model of research, such as Alexander et al [7] approaches such as the use of controlled speed of sound; Yu and Zhao [4] introduce magnetism to reduce the speed of sound, thus alleviating the constraints of small Mach number impact, but these methods do not restore the energy equation, and apply to a limited extent. Palmer and Rector et al [9, 10] made the thermal LBM model, but still do not reflect the high Mach number phenomenon; Qu et al [7] proposed to use a circle function instead of using the Maxwell distribution function; Li et al [8] proposed a pairs of distribution function method; Sun et al [9] made the locally adaptive LB model, the speed of his model can get a very wide speed unlimited size; Yan Guang-wu [10] proposed multi-level multi-speed compressible model. These methods are the methods of changing the model of LBM proceed to find a distribution function and lattice model of a suitable compressible problem, in order to solve compressible fluid flow problems. These models themselves are some shortcomings, such as some models require large amount of computing, and some itself is complex, and some lack of rigorous theoretical derivation, and some extended to three-dimensional problem more difficult, reducing the usefulness.

Recently, Shan Xiao-wen et al [11] introduced Hermite functions and derived distribution function, and made theoretically the LBM method for solving compressible fluid flow problems, this method has a more rigorous mathematical derivation. This paper will use this method to study high speed compressible fluid problems.

2. The Lattice Boltzmann Method

Qian [2, 3, 4] and Chen [5, 6] et al used independently Bhatnagar-Gross-Krook (BGK) collision relaxation model, and proposed lattice BGK (Lattice BGK, LBGK) model, the complex collision operation transformed into a simple relaxation process:

$$f_i(\mathbf{x}+\mathbf{e_i}, t+1) = (1-\omega) f_i(\mathbf{x}, t) + \omega f_i^{eq}(\mathbf{x}, t), \tag{1}$$

where $f_i(\mathbf{x}, t)$ is defined in the discrete velocity set $\mathbf{e_i}$, is particle density distribution function at time t in the space grid points $\mathbf{x}$, $f_i^{eq}(\mathbf{x}, t)$ is the amount by the system's current macro-constructed local dynamic equilibrium distribution function, relaxation factor ω depends on the physical properties (such as fluid viscosity, thermal conductivity coefficient, mass diffusion coefficient). In LBGK model, The macrosystem state is constructed by the micro-particle group, on the other hand, the micro-particle group movement is controlled by the local macro-state of dynamics system. This reflects the individual partial discipline and adaptability. The