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NUMERICAL TREATMENT OF TWO-PHASE FLOW IN CAPILLARY HETEROGENEOUS POROUS MEDIA BY FINITE-VOLUME APPROXIMATIONS

HELMER ANDRÉ FRIIS AND STEINAR EVJE

This paper is dedicated to the memory of Professor Magne S. Espedal (1942-2010)

Abstract. This paper examines two-phase flow in porous media with heterogeneous capillary pressure functions. This problem has received very little attention in the literature, and constitutes a challenge for numerical discretization, since saturation discontinuities arise at the interface between the different homogeneous regions in the domain. As a motivation we first consider a one-dimensional model problem, for which a semi-analytical solution is known, and examine some different finite-volume approximations. A standard scheme based on harmonic averaging of the absolute permeability, and which possesses the important property of being pressure continuous at the discrete level, is found to converge and gives the best numerical results. In order to investigate two-dimensional flow phenomena by a robust and accurate numerical scheme, a recent multi point flux approximation scheme, which is also pressure continuous at the discrete level, is then extended to account for two-phase flow, and is used to discretize the two-phase flow pressure equation in a fractional flow formulation well suited for capillary heterogenity. The corresponding saturation equation is discretized by a second-order central upwind scheme. Some numerical examples are presented in order to illustrate the significance of capillary pressure heterogeneity in two-dimensional two-phase flow, using both structured quadrilateral and unstructured triangular grids.

Key words. two-phase flow, heterogeneous media, capillary pressure, finite volume, MPFA, unstructured grids

1. Introduction

The study of two-phase flow in porous media has significant applications in areas such as hydrology and petroleum reservoir engineering. The flow pattern is mainly governed by the geometric distribution of absolute permeability, which may be anisotropic and highly heterogeneous, the form of the relative permeability and capillary pressure functions and gravity [4]. The corresponding system of partial differential equations describing the flow consists of an elliptic and an essentially hyperbolic part, usually denoted the pressure- and saturation equation, respectively. This system is rather challenging, and quite a lot of research has been devoted to its solution during the last decades.

In recent years several discretization methods that can treat unstructured grids in combination with discontinuous and anisotropic permeability fields have been developed for the elliptic pressure equation. Important examples are the flux-continuous finite volume schemes introduced in e.g. [11, 12, 25, 13, 15, 5, 6], which have been termed multi point flux approximation methods (MPFA) schemes, and the mixed finite element (MFE) and related schemes, e.g., [1, 2, 9, 20, 18]. The MFE and related methods solve for both control-volume pressure and cell face velocities leading to a globally coupled indefinite linear system (saddle point problem), while the more efficient MPFA methods only solve for control-volume pressure and have

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a locally coupled algebraic system for the fluxes that yield a consistent continuous approximation, while only requiring one third the number of degrees of freedom of the mixed method when compared on a structured grid (and a quarter in three dimensions). The latter methods are clearly advantageous, particularly for time-dependent problems, as the extra degrees of freedom required by the mixed method add further computational complexity and a severe penalty to simulation costs. For the saturation equation some higher order schemes have been employed, as well as various types of so called fast tracking schemes, but the standard first order upwind scheme is still widely used in commercial simulators.

However, the main body of research literature devoted to two-phase flow in porous media concerns flow in the absence of capillary pressure, or assumes a homogeneous capillary pressure function in the domain. Obviously, there are a number of flow cases for which these assumptions are valid, but this observation is nevertheless noticeable since heterogeneity in capillary pressure may often have a significant effect on the flow pattern, and in certain cases it can be as important as absolute permeability heterogeneity [19].

From the very sparse literature devoted to capillary pressure heterogeneity in porous media, we would like to mention the work of Yortsos and Chang [27]. They studied analytically the capillary effect in steady-state flow in one-dimensional (1D) porous media. They assumed a sharp, but continuous transition of permeability to connect different permeable media of constant permeabilities. The paper by van Duijn and de Neef [26] on the other hand, provided a semi-analytical solution for time-dependent countercurrent flow in 1D heterogeneous media with one discontinuity in permeability and capillary pressure. Niessner et al. [24] discuss the performance of some fully implicit vertex-centered finite volume schemes, when implementing the appropriate interface condition for capillary heterogeneous media. The recent paper by Hoteit and Firoozabadi [19] presents an MFE method for discretising the pressure equation together with a discontinuous Galerkin method for the saturation equation. They introduced a new fractional flow formulation for two-phase flow, which is suited for applying MFE in media with heterogeneous capillary pressure. Some numerical examples are presented, including a comparison with the 1D semi-analytical solution from [26], demonstrating good performance of the numerical scheme.

The simulation of two-phase flow in porous media with capillary pressure heterogeneity represents a challenge for the actual numerical discretization. This is particularly due to the fact that saturation discontinuities arise at the interface between the different homogeneous regions of the domain, as a result of the requirement of capillary pressure continuity. Moreover, since these are rather involved nonlinear problems, very few analytical results are known, making it more difficult to gain confidence in the results produced by the numerical schemes. Clearly, as discussed in [26], the capillary pressure may also actually become discontinuous at the interface in some situations. This depends on the form of the capillary pressure curve (the entry pressure) together with the actual type of two-phase flow in the problem. In the more usual situations where a wetting phase is displacing a non-wetting phase, this phenomenon will not occur. Moreover, since this particular situation does not introduce any new fundamental issues with respect to the numerical treatment of these problems, we only consider examples with capillary pressure continuity at the interface in this paper.

As noted in [19] MPFA methods have not yet been demonstrated to be of value for heterogeneous media with contrast in capillary pressure functions. This fact