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TENSOR RELATIVE PERMEABILITIES: ORIGINS, MODELING AND NUMERICAL DISCRETIZATION

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This paper is dedicated to the memory of Professor Magne S. Espedal

Abstract. For multi-phase flow in porous media, the fluids will experience reduced conductivity due to fluid-fluid interactions. This effect is modeled by the so-called relative permeability. The relative permeability is commonly assumed to be a scalar quantity, even though justifications for this modeling choice seldom are presented. In this paper, we show that the relative permeability can yield preferential flow directions, and that furthermore, these directions may vary as a function of the fluids present. To model these effects properly, the relative permeability should be considered a tensor. As shown in the paper, standard numerical methods cannot simulate tensor relative permeabilities. We therefore propose two new methods to remedy the situation. One scheme can be seen as an attempt to amend traditional methods to handle tensor relative permeabilities. The second scheme is a consistent approach to simulate the tensors. We also present numerical simulations of cases where upscaling leads to tensor relative permeabilities. The results show that the new methods are indeed capable of capturing tensor effects, moreover, they are in very good agreement with fine scale reference solutions.

Key words. porous media, multi-phase flow, tensorial relative permeability, directional dependency, MPFA, discontinuous Riemann problems

1. Introduction

Multi-phase flow in porous media is the governing physics for many natural and artificially constructed systems. Natural systems of interest are of both geologic and biologic character, while artificial systems pertain primarily to engineered materials. Controlling fluid flow in complex porous media is consequently of great practical importance. While single-phase flow is fundamentally well understood, flow of multiple fluid phases still provides challenges for modeling and simulation. Herein we address the challenge when the material has a preferential direction of conductance, and when this direction may be fluid fraction dependent at the modeling scale.

It is common in experiments and modeling to consider the anisotropic nature of intrinsic permeability. This is understood physically as resulting from subscale structuring of the porous media itself. In both natural and artificial porous media the finest scale at which Darcy's law is applicable is often referred to as the Representative Elementary Volume (REV). However, even the REV scale will often contain structure on the finer scale which is the pore-space itself. When this structure is expressed as layering, or channeling, fluid flow will be preferential in some directions, which is parameterized as an anisotropic permeability. Textbook examples are wood (channeling) and geological media (layered at all scales due to compaction).

It is natural to assume that this subscale structure will also impact the fluid distribution when two or more fluids are present simultaneously in a porous media. Such a subscale structure in fluid distribution may well alter the effective anisotropy.

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The multi-fluid flow properties of porous media can be parameterized by the so-called relative permeability functions. From our discussion, we are tempted to state that these relative permeability functions are anisotropic, with anisotropy ratio and direction that may both be functions of saturation. However, common modeling almost universally neglects this aspect, with the a-priori assumption that the anisotropy ratio is independent of saturation.

Several previous studies have shown that the relative permeability function may be anisotropic at scales ranging from the REV scale [12], through the laboratory scale [15], to the field scale [26, 27, 24]. Furthermore, in these studies it is frequently observed that a full description of relative permeability give significant qualitative differences from scalar models.

The aim of this contribution is therefore two-fold: Firstly, to highlight mechanisms that cause saturation-dependent anisotropy at all scales. Secondly, to propose numerical approaches that adapt to the significant challenges this saturation dependence causes in practical modeling.

In this paper, we begin to address both these points. In Section 2, we discuss the understanding that exists regarding relative permeability (in the continuation, we will always imply that relative permeability is anisotropic, unless otherwise stated). In particular, we show using a network flow model how anisotropy arises even at the finest continuum scale, and subsequently consider the emergence of anisotropy in relative permeability from upscaling procedures. In Section 3, we highlight some of the main challenges that arise from a simulation perspective with a full description of relative permeability, using IMPES time-stepping with control-volume spatial discretization as a backdrop for our discussion. Acknowleding these difficulties, in Section 4 we propose ideas for handling the full tensors within the traditional numerical framework, as well as an approach that allows for the saturation equation to be solved consistently. Section 5 combines the work of the preceding sections within numerical implementations. These illustrate A) Flow patterns that are unique to full relative permeability systems and B) The performance of our proposed solution approaches. The paper is concluded in Section 6.

2. The nature of relative permeability

While the permeability of a porous material is almost universally modeled as anisotropic, relative permeability is consistently modeled as a scalar function of saturation (see e.g. [20, 11]). Notable exceptions exist (see e.g. [12, 26, 27, 24, 10]), however these references usually consider anisotropy in relative permeability as a consequence of upscaling, rather than intrinsic to the porous media itself. In this section, we will argue that relative permeability should be modeled as anisotropic at all scales, by considering firstly the finest scale at which Darcys law is applied (the REV scale). Subsequently, we will consider anisotropic relative permeability as it arises from upscaling vertically segregated systems.

To be concrete, we establish the following definitions relating to permeability:

- The intrinsic permeability $\mathbf{K}$ is the permeability of a porous medium saturated by one fluid.
- The effective permeability $\mathbf{K}_\alpha$, with respect to a fluid phase α, is the permeability of a porous medium as experienced by that fluid.
- The relative permeability $\mathbf{K}_{r,\alpha}$, is the ratio of the effective to the intrinsic permeability. In the case where the two latter are tensors, the relative permeability can be defined as $\mathbf{K}_{r,\alpha} = \mathbf{K}_\alpha \cdot \mathbf{K}^{-1}$.