INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 3, Number 2, Pages 192–206

AN ENHANCED MATHEMATICAL MODEL FOR PHASE CHANGE PROBLEMS WITH NATURAL CONVECTION

YOUSSEF BELHAMADIA, ABDOULAYE S. KANE, AND ANDRÉ FORTIN

Abstract. The enthalpy-porosity technique is commonly used for modeling phase change problems with natural convection. In most applications however, this technique is restricted to equal thermophysical properties of solid and liquid phases. In this paper, an enhanced formulation based on the enthalpy-porosity method is proposed where the different thermophysical properties between the two phases can be easily taken into account. Accurate temporal and spatial discretizations are also presented for solving the proposed formulation. Numerical simulations on pure gallium melting and a comparison between experimental and numerical results for water solidification are presented to illustrate the performance of the proposed model and numerical methodology.

Key words. Phase change problems with natural convection, mixed finite element formulation, interface liquid-solid, gallium melting, and water solidification

1. Introduction

Phase change problems with natural convection play a significant role in several industrial applications. The main challenge of such problems is the presence of a moving liquid-solid interface involving a strong coupling of mass and heat transfer. The mathematical models for solving phase change with convection can be roughly divided into two categories. The first one is based on a multi-domain approach where the momentum and energy equations are solved in each phase domain separately (see Florez et al. [10], Gupta [14], Viswanath and Jaluria [23], and the references therein). This approach thus requires a continuous update of the two domains due to the time dependent interface position. The second category is based on a single-domain approach where a system of momentum and energy equations is solved in the entire physical domain (see Rady and Mohanty [19], Samarskii et al. [20], Voller and Prakash [25], and the references therein). The main advantage of this approach is that the interface is not explicitly computed and the energy balance condition is automatically satisfied at the interface.

The enthalpy-porosity model is widely used as a single domain approach (see Brent et al. [5] and Voller et al. [24]). This model includes latent heat effects as a source term and includes a technique to ensure that the velocity field vanishes in the solid region. This model is however mostly used when the thermophysical properties of solid and liquid are equal (see Hannoun et al. [16, 15] and Evans et al. [9], Evans and Knoll [8]). In the more general case, the enthalpy-porosity model can be reformulated into a vorticity-velocity model. This formulation is based on averaging the physical variables velocity, density, and thermal conductivity, by using liquid-solid mass and volume fraction. The reader is referred to Kowalewski and Rebow [18], and Giangi et al. [12, 13] for a more detailed description.

Phase change problems including natural convection are challenging from a numerical point of view. Small time steps and very fine meshes as well as accurate space and time discretizations are required if one wants to capture the complex

Received by the editors February 7, 2012 and, in revised form May 20, 2012.
2000 *Mathematics Subject Classification.* 80A20, 80A22, 80M10, 65M60, 76D05.

physics of the problem and especially to accurately compute the liquid-solid interface where phase change occurs. Indeed, the use of second order accurate spatial discretizations on tin and gallium melting problems have been investigated in Hannoun et al. [16] and it is concluded that second order accuracy as well as fine meshes are required to obtain the proper number and location of roll cells. The mesh dependence of the flow structure of pure gallium melting was put in evidence in Stella and Giangi [21]. However, only first order fully implicit time discretizations have been tested in these previous works. To our knowledge, the issue of time discretization accuracy for phase change convection simulations has been addressed only in Evans and Knoll [8] where temporal accuracy analysis using a Jacobian-free Newton-Krylov (JFNK) solution method with a pressure-correction smoother (SIMPLE) algorithm is performed for the non-dimensional solidification test case and pure gallium melting problems.

Phase change problems have enormous importance in many engineering and industrial processes. Freezing of water is an example that has recently received a lot of experimental and numerical attention. In Kowalewski and Rebow [17, 18], an experimental benchmark was set up to study the transient natural convection during freezing of water in a cube-shaped cavity. The experimental data was compared to numerical results performed with a finite difference method and using the multi-domain approach where the governing equations are solved separately for the fluid and solid domain. Important discrepancies were observed in the interface position and flow patterns. In Giangi et al. [12, 13] numerical and experimental results were also presented based on a similar freezing of water problem. This time, the numerical results were obtained using a finite volume method and a vorticity-velocity model. Discrepancies between numerical and experimental results were again reported despite the fact that the mesh dependance and the effects of thermal boundary conditions were closely analyzed. Similar discrepancies for water solidification has been also reported in Banaszek et al. [1]. A semi-implicit finite element method was adopted for the two-dimensional simulation and the mathematical model was based on the general enthalpy formula as presented in Swaminathan and Voller [22].

In a previous work of the authors, a finite element formulation has been introduced for phase change problems without convection. This formulation considers both equal and different liquid-solid physical properties and has been validated on various two and three dimensional problems (see Belhamadia et al. [2, 3], Fortin and Belhamadia [11]). The main goal of this work is first to present a more general form of this formulation in order to include the presence of natural convection. The proposed formulation is based on an enthalpy-porosity model and allows us to consider the case where liquid and solid physical properties differs. A fully implicit finite element mixed formulation for both the momentum and energy equations is also proposed. The discretization is second order accurate in both space and time. The advantage and performance of the overall technique are analyzed on two classical benchmark problems: pure gallium melting where our numerical solutions are compared with the existing literature results and water solidification where a comparison between experimental and numerical results is performed showing that the above mentioned discrepancies are clearly reduced when using the proposed method.

The organization of this paper is as follows. In Section II, the enthalpy-porosity model is briefly recalled and the proposed model is introduced. Section III is devoted to the numerical method and finite element discretization. Finally, section IV presents numerical results showing the performance of the proposed method.