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NUMERICAL SOLUTIONS OF A HYPERSINGULAR INTEGRAL EQUATION WITH APPLICATION TO PRODUCTIVITY FORMULAE OF HORIZONTAL WELLS PRODUCING AT CONSTANT WELLBORE PRESSURE

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Abstract. The performance of horizontal wells producing at constant wellbore pressure is a critical problem in petroleum engineering. But few articles on the well performance under constant wellbore pressure can be found in the literature due to the difficulty of hypersingular integral equations, which are needed for this problem. This article proposes and studies a new model using a hypersingular integral equation for the productivity of horizontal wells producing at constant wellbore pressure. An efficient numerical method is developed for this hypersingular integral equation based on a new expansion with respect to the singularity at arbitrary points. And numerical examples are provided to illustrate the convergence of the numerical methods. By using fluid potential superposition principle, productivity equations for a line sink model are derived from the point sink solution to the diffusivity equation. By solving the hypersingular integral equation, the authors obtain the productivity formulae of a horizontal well producing at constant wellbore pressure, which provide fast analytical tools to evaluate production performance of horizontal wells. Numerical examples are provided to illustrate the features of the model and the numerical method.

Key words. Hypersingular Integral Equation, Quadrature method, Horizontal Well, Constant Wellbore Pressure.

1. Introduction

For horizontal wells, steady-state and unsteady-state pressure transient testings are critical tools to evaluate in-situ reservoir and wellbore parameters that describe the production characteristics of a well. The use of transient well testings for determining reservoir parameters and productivity of horizontal wells has become common because of the upsurge in horizontal drilling. Hence the determination of transient pressure behavior and productivity for horizontal wells has aroused considerable interest recently [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

During the last two decades, numerous analytic solutions have been presented for the pressure behavior of horizontal wells producing at constant flow rates. Interpretation of well tests from horizontal wells is much more difficult than interpretation of those from vertical wells due to a considerable wellbore storage effect, the three dimensional nature of the flow geometry, the lack of radial symmetry, and the strong correlations between certain parameters, see [3, 4, 5, 6, 7, 8, 9, 10] and references therein. Joshi [11] presented a steady state productivity formula for horizontal wells based on a pseudo three-dimensional reservoir model which was splitted into two two-dimensional models. Babu [12] presented a pseudo-steady state productivity formula for a horizontal wells in a closed box-shaped reservoir. By solving three dimensional Laplace equation and using fluid potential superposition principle, Lu [13, 14, 15, 16] presented steady state productivity formulae

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and pressure transient formulae of horizontal wells in infinite slab reservoirs and circular cylinder reservoirs.

Most well test analysis methods assume constant rate production since constant wellbore pressure production conditions are common. Examples of conditions under which constant pressure is maintained at a well include production into a constant pressure separator or pipeline, open flow to the atmosphere, and production from a low permeability reservoir. Particularly, in order to keep a steady water cone, the constant wellbore pressure production is required for a reservoir with bottom water.

In order to solve the problem of constant wellbore pressure production, one often needs the solution to a hypersingular integral equation [17]. In recent years, different types of singular integral equations have been utilized to study the problems in acoustics, fracture mechanics, elastic mechanics, electromagnetics, traffic flow, and so on [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. And different efficient numerical methods for hypersingular integral equations have been also studied, such as Galerkin methods [33, 34, 35, 36, 37, 38], quadrature methods [39, 40, 41], additive Schwarz method [42, 43, 44], multilevel methods [42, 45], collocation methods [46, 47], Newton's method [48], hybrid method [49] and others [50, 51, 52, 53]. See [54] and references therein for a survey on the hypersingular integral equations.

This article proposes and studies a new model, which uses the hypersingular integral equation (1) for the productivity of horizontal wells producing at constant wellbore pressure. To our best knowledge, there are few papers discussing the numeric solution of this type of hypersingular integral equation even though a number of numerical methods have been developed to solve other types of hypersingular integral equations. Galerkin method can be used to solve equation (1). But the computational cost is relatively high. Without non-trivial extension, other existing methods may not be directly valid for equation (1) due to the difficulties caused by the term $2\psi\left(\frac{1}{2}\right) - 2\ln\left(\frac{1}{h}\right)$ in (28).

In this article, an efficient numerical method is proposed to solve the equation (1) based on a new expansion with respect to the singularity at arbitrary points instead of the endpoints and then the results are applied in the new productivity model of horizontal wells producing at constant wellbore pressure. The hypersingular integral equation is studied in Section 2 and the numerical method for this equation is developed in Section 3. In Section 4, three numerical examples are presented to illustrate the convergence of this proposed method and solve the hypersingular integral equation arising from the new model. In Section 5, the new model is presented and the numerical solution of the hypersingular integral equation is used to obtain a universal productivity formula of horizontal wells producing at constant wellbore pressure.

2. A hypersingular integral equation for the new model

In order to obtain the productivity formulae of horizontal wells under the constant wellbore pressure in Section 5, we first consider the following new model of an integral equation which will be derived in Section 5:

$$\int_{-1}^{1} \frac{g(x)}{|x-y|} dx = -1, \forall y \in [-1, 1], \tag{1}$$

where the unknown function $g(x)$ is called the point convergence intensity in petroleum engineering.