INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 5, Number 4, Pages 317–338

IMPROVING NUMERICAL ACCURACY IN A REGULARIZED BAROTROPIC VORTICITY MODEL OF GEOPHYSICAL FLOW

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Abstract. We study the BV-α-Deconvolution model. It is a family of regularizations of the Barotropic Vorticity (BV) model that generalize the BV-α model and improve its accuracy. A both unconditionally stable and optimally convergent scheme for the BV-α-Deconvolution model is proposed and we show that it is $O(\alpha^{2N+2})$, where N is the deconvolution order, whereas the BV-α model is at most second order accurate. We perform numerical simulations to confirm the predicted convergence rates and test the model in the traditional double gyre wind experiment. For the latter test, we show that the BV-α-Deconvolution model can retrieve the expected high resolution pattern being more accurate for larger values of deconvolution order.

Key words. Barotropic vorticity model, regularizations, turbulence, geophysical flow.

1. Introduction

Accurate simulations of geophysical flows are critically important in understanding climate change and ocean and weather forecast. Furthermore, they can assist in prediction of biological and pollutant transport, oil exploration, and many other applications. One of the simplest nonlinear models to simulate a geophysical flow is the Barotropic Vorticity (BV) model, which, in dimensionless, form is given by [8, 22]

$$\text{(1a)} \qquad Ro\,\frac{\partial \omega}{\partial t} + Ro\,J(\psi,\omega) - \frac{\partial \psi}{\partial x} - \left(\frac{\delta_M}{L}\right)^3 \Delta\omega = \mathcal{F},$$

$$\text{(1b)} \qquad \Delta\psi = -\omega,$$

where ω is the vorticity, ψ is the streamfunction, $J(\psi,\omega) = \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x}$ is the Jacobian, Ro is the Rossby number, δ_M is the Munk scale, L is the length scale and $\mathcal{F}$ is the forcing term. The BV model is widely used to study the midlatitude, wind-driven ocean circulation, and it recently has been used in studies involving data assimilation [27, 4, 9], climate [19, 14, 5] and oceanic and atmosphere processes [3, 25, 20].

Despite its simplicity, the BV model is very sensitive to the mesh resolution [6, 16, 8, 22], making full representation of the solution computationally expensive. This becomes critical when long time integration is necessary, as in climate modeling. Traditionally, simulations are done on coarse meshes and (essentially) dissipative techniques such as eddy viscosity parametrizations have been used to model the under-resolved scales of the flow. However, according to [8], increasing artificial viscosity tends to reduce variability, and nonlinear structures can be destroyed by excess of dissipation [6, 8]. Thus some methods such as Approximate Deconvolution Modeling [22, 21], Barotropic Vorticity-α (BV-α) [17, 16, 8] and

Received by the editors July 18, 2014 and, in revised form, September 18, 2014.

2000 *Mathematics Subject Classification.* 65M12,65M60,76D99,76U99.

Igor O. Monteiro is partially supported by PRH-PB16. Carolina C. Manica is partially supported by CNPQ, 480525/2011-0.

Barotropic Vorticity-Bardina [10] have been developed with success to improve accuracy and reduce the degrees of freedom in computational simulations. The BV-α model is a regularization of the BV model proposed in [8] that allows a significant reduction of degrees of freedom in simulations. In BV-α, the nonlinearity is altered so that the flow at length scales that are smaller than the alpha length scale are nonlinearly removed by motions at the larger scales. Thus there is seemingly no need to introduce additional dissipative terms or increase the viscosity coefficient, which is often done in the BV equations. The BV-α model is given by

$$\begin{aligned} Ro\,\frac{\partial \omega}{\partial t} + Ro\,J(\psi,\omega) - \frac{\partial \psi}{\partial x} - \left(\frac{\delta_M}{L}\right)^3 \Delta\omega &= \mathcal{F}, && \text{(2a)}\\ \Delta\psi &= -\overline{\omega}, && \text{(2b)}\\ -\alpha^2\Delta\overline{\omega} + \overline{\omega} &= \omega, && \text{(2c)} \end{aligned}$$

where $\overline{\omega}$ is the filtered vorticity and α is the filter length scale. A more complete description of BV-α is presented in [8].

Despite being physically accurate, the BV-α model naturally has a consistency error from the BV model. It is clear from (2) that one cannot expect accuracy better than $O(\alpha^2)$. Since frequently $\alpha = O(h)$, the BV-α model is often only second order accurate. Following [15], we attempt to fix the consistency error in the BV-α model by increasing its accuracy through the van Cittert method of approximate deconvolution [24, 1, 23]. The method constructs a family D_N of approximate inverses to the filter F as the truncation of the nonconvergent formal power series $F^{-1} = \sum_{n=0}^{\infty}(I-F)^n$:

$$D_N = \sum_{n=0}^{N}(I-F)^n. \tag{3}$$

In [18], it is shown how to apply the deconvolution operator in the Navier-Stokes-α to achieve accuracy $O(\alpha^{2N+2})$, where N is the order of deconvolution. Thus we adopt the above mentioned approach and introduce the BV-α model with deconvolution (BV-α-Deconvolution) by

$$\begin{aligned} Ro\,\frac{\partial \omega}{\partial t} + Ro\,J(\psi,\omega) - \frac{\partial \psi}{\partial x} - \left(\frac{\delta_M}{L}\right)^3 \Delta\omega &= \mathcal{F}, && \text{(4a)}\\ \Delta\psi &= -D_N\overline{\omega}, && \text{(4b)}\\ -\alpha^2\Delta\overline{\omega} + \overline{\omega} &= \omega. && \text{(4c)} \end{aligned}$$

The BV-α-Deconvolution model, as we will show in the next sections, will allows a reduction of degrees of freedom in simulations but with a consistency error $O(\alpha^{2N+2})$ when compared to the BV model.

The paper is organized as follows: Section 2 introduces a finite element scheme for BV-α-Deconvolution and some necessary notation and mathematical preliminaries. Section 3 presents the stability analysis of the proposed scheme. Section 4 presents the convergence analysis. Convergence rates are estimated and the double gyre experiment is performed in Section 5. Finally, some conclusions and remarks are summarized in Section 6.