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Entanglement dynamics of two trapped ions in the intermediate excitation regime

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Abstract. We have studied the model for interaction of two trapped ions in a trap with a laser beam in the intermediate excitation regime. By applying unitary transformations, the system can be transformed into the Tavis-Cummings model. The entanglement dynamics of two trapped ions in this system has been investigated. With computation of the concurrence, unlike that in Tavis-Cummings model, we find that the entanglement of two ions in our model undergoes periodic death and revivals.

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Key words: trapped ion, entanglement sudden death, concurrence

1 Introduction

The entanglement in quantum systems has received great attentation not only because it is of fundamental interest in quantum mechanics but also because it plays very important role in processing of quantum information such as quantum cryptography [1], quantum key distribution [2], quantum superdense coding [3], quantum teleportation [4] and quantum computation [5]. The entanglement of two particles has been demonstrated experimentally using ultra cold trapped ions [6] and cavity quantum electrodynamics schemes [7].However, one has found that the entanglement of the bipartite system being open to environment will be decay exponentially as similar as quantum decoherence. This kind of decay is called as the loss of entanglement. Quite recently, one found that the entanglement between the bipartite coupled with independent reservoirs terminates abruptly in finite time, which is called the entanglement sudden death (ESD). It has been shown that ESD effect is not only sensitive to the initial states of systems, but also is dependent on property of noise [8–11]. For a special initial state, the entanglement of two particles will disappear and then revive.

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There is growing interest in the entanglement of trapped ions. On the one hand, it has been realized that the trapped ions can be located at a known and controllable distance from one another. On the other hand, it was discovered that the trapped ions can be prepared in maximally entangled states that are isolated from its environment [12]. In Ref. [13], the authors show that ESD can be influenced by the Stark shift and long-lived entanglement can be produced between the two trapped ions that are driven by a classical light field in the weak excitation region.

In this paper, we investigate the dynamics of the entanglement between two trapped ions in the intermediate excitation regime. We find that the entanglement of two ions undergoes periodic death and revives. An explicit connection between the entanglement dynamics of two trapped ions and their initial states is presented.

2 Theoretical model

We consider two two-level ions trapped in a harmonic potential traps, which interacts with a laser field with frequency ω.We restrict our consideration to the quantum mechanical motion of the ion in the x direction and omit the breathe mode of the ions. The Hamiltonian of system is given by [14]

$$H=H_t+\hbar\frac{\Delta}{2}J_z+\frac{\hbar\Omega}{2}(e^{i\eta(a+a^+)}J_++e^{i\eta(a+a^+)}J_-), \tag{1}$$

where $H_t=\hbar\nu a^+a$,

$$J_z=\sum_{i=1}^{2}\sigma_z^{(i)},\ J_+=\sum_{i=1}^{2}\sigma_+^{(i)},\ J_-=\sum_{i=1}^{2}\sigma_-^{(i)}$$

and $a(a^+)$ is the annihilation (creation) operator of the center-of-mass vibrational mode of the ion, $\Delta=\omega_a-\omega$ is the detuning of the ionic transition from the laser frequency, which is set $\Delta=0$, the Lamb-Dick parameter $\eta=\pi a_0/\lambda$, with a_0 the amplitude of the ground state of the trap and the optical wavelength, $\sigma_z^{(i)}$, $\sigma_+^{(i)}$ and $\sigma_-^{(i)}$ are pseudospin inversion, raising, lowering operators of the ith ion and set $\eta\ll 1$. Applying the Lamb-Dicke approximation, neglecting the higher order terms on η, Eq.(1) can be represented in the form of the matrix as ($\hbar=1$)

$$H=H_t+\hbar\frac{\Delta}{2}J_z+\frac{\Omega}{2}J_x-\frac{\eta^2\Omega}{4}(a+a^+)^2J_x+\frac{\eta\Omega}{2}(a+a^+)J_y, \tag{2}$$

where $J_x=J^++J^-$, $J_y=-i(J^++J^-)$. In order to investigate entanglement dynamics of two trapped ions in the intermediate excitation regime ($\Omega\square\nu$), we cannot make directly rotating wave approximation. We firstly make a unitary transformation $H'=THT^+=e^{i\pi J_y/4}He^{-i\pi J_y/4}$. The transformed Hamiltonian reads

$$H'=H_t+\frac{\Omega}{2}J_z-\frac{\eta^2\Omega}{4}(a+a^+)^2J_z-i\frac{\eta\Omega}{2}(a+a^+)(J^+-J^-). \tag{3}$$