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Bound state solutions of exponential-coshine screened coulomb plus morse potential

Akpan N. Ikot[a,*], Oladunjoye A. Awoga[a], and Benedict I.Ita[b]

[a] *Theoretical Physics Group, Department of Physics, University of Uyo, Nigeria*
[b] *Theoretical Quantum Chemistry Group, Department of Chemistry, University of Calabar, Nigeria*

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Abstract. The analytical solutions of the Schrödinger equation with exponential coshine screened plus Morse (ECSM) potential are presented. The energy eigenvalues and the corresponding eigenfunctions are obtained for several values of screening parameters. We also present some numerical results for some selected diatomic molecules which are consistent with the values in the literature.

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Key words: Nikiforov-Uvarov method, energy eigenvalues, eigenfunctions diatomic molecules

1 Introduction

The bound state solutions of the Schrödinger equation (SE) are only possible for some potentials of physical interest [1–4]. The exact or approximate solutions contain all the necessary information for the quantum system. In recent years, the problem of exact or approximate solution of the Schrödinger equation for a number of special potentials has been of great interest [5–7]. Different authors have obtained the exact solutions of the SE with some typical potentials using various methods. These potentials include the harmonic oscillator potential [8], Eckart potential [9], Woods-Saxon potential [10], Pseudo-harmonic oscillator potential [11], ring-shaped Kratzer-type potential [12], ring-shaped non-spherical oscillator potential [13], Rosen - Morse potential [14], Hulthen potential [15], etc. Different methods have also been employed in the study of those special potentials: Asymptotic Iteration Method (AIM) [16,17], Exact Method (EM) [18], Shifted $\frac{1}{N}$ Expansion [19], Super Symmetric Quantum Mechanics (SUSYM) [20], Tridiagonal J - Matrix [21], Algebraic Method [22] and Nikiforov - Uvarov (NU) Method [23].

*Corresponding author. *Email address:* `ndemikot2005@yahoo.com (A. N. Ikot)`

Some of these potentials are known to play important roles in many fields of Physics such as molecular Physics, solid state Physics and chemical Physics [24]. One of such potential is the Morse potential and its vibrational energy levels are obtained using various techniques [25]. It has been reported that the potential energy function for the lowest electronic states of diatomic molecules can be expressed by the Morse Potential [25] as

$$V(r)=D_e[1-e^{-\alpha(r-r_0)]^2}]^2, \tag{1}$$

Where D_e is the dissociated energy, α is adjustable parameter, r is the inter-atomic separation distance and r_0 is the distance from equilibrium position. Another well studied potential is the generalized exponential-cosine-screened coulomb (GECSC) potential defined as [26]

$$V(r)=-\frac{A}{r}e^{-\delta r}\cos(g\delta r), \tag{2}$$

Where A is the coupling strength constant, δ is the screening parameter, $g=1$ is the cosine-screened coulomb constant. The special case of the GECSC potential known as the static screened coulomb (SSC) takes the form: $V(r)=-(\alpha Ze^2)\exp(-\delta r)/r$ with $A=\alpha Ze^2$, α is the fine structure constant and Z is the atomic number and has been used in the description of the energy levels of light and heavy neutral atoms [26].

The exact solution for any $l\neq 0$ for the GECSC potential is still unknown. However, approximate methods [26] have been developed to evaluate the numerical and analytical values of its bound state.

For the purpose of this study, we investigate the Exponential Coshine Screened plus Morse (ECSPM) potential. This ECSPM potential is defined as

$$V(r)=D_e[e^{-2\delta r}-2e^{-\delta r}]-D_e\cosh e^{-\delta r}, \tag{3}$$

Where δ is the screening parameter and D_e is the dissociation energy. This potential can be used for the description of diatomic molecular vibrations [27]. They can also be useful in many branches of Physics for their bound and scattered problems [26]. Taking the first derivative of Eq. (3), that is, $(\frac{dV(r)}{dr})_{r=r_0}=0$ we get,

$$V(r_0)=\frac{(1-D_e\delta/D_e)}{(1+D_e\delta/2D_e)}-\frac{D_e\delta}{2}, \tag{4}$$

and

$$r_0=\frac{1}{\delta}\ln(1+\frac{D_e\delta}{2D_e}),\quad \text{for}\quad \delta>0. \tag{5}$$

It can be observed that $r_0=1$ for $\delta\ll 1$ and $V(r_0)=-D_e-D_e\delta/2$. Similar observation can also be made for the following cases: (i) $D_e\to 0$ and (ii) $D_e\delta\to 0$ and the second derivative of the potential at $r=r_0$ determines force constant.

Satisfied with its performance through comparison with other methods, we decided to apply NU method to solve the ECSPM potential. Moreover, this work will show that