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The effect of temperatures and magnetic field on relativistic Fermi gas

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Abstract. By using the methods of theoretical analysis and numerical simulation, the relativistic effects of the thermodynamic properties of Fermi gas in a strong magnetic field are studied, and the influential mechanisms of temperatures and the magnetic field on the relativistic effects of the system are analyzed. It is shown that, at low temperatures, the effects of temperatures on the relativistic effect relate to the chemical potential of the free system. The magnetic field's effects on the relativistic effects of heat capacity and chemical potential display oscillating characteristics. At high temperatures, the relativistic effects of the thermodynamic quantities trigger oscillations with increasing of temperatures and magnetic field, and its amplitude decreases obviously to disappearing with the increasing of temperatures. However, its amplitude increases evidently due to the amplified magnetic field.

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Key words: Fermi gas, relativity, magnetic field, temperature

1 Introduction

Recently, researches on ultra-cold Fermi gases have obtained abundant achievements [1-8], and the studies of relativity Fermi gas have been a hot field due to the discovery of neutrino. Under the condition of considering the relativistic effect, we have investigated the thermodynamic properties and stability of a weakly interacting Fermi gas in a weak magnetic field [9,10] researched weakly interacting Fermi gas's relativistic paramagnetism [11], and studied relativistic thermodynamic properties of Fermi gas in hard -sphere potential by using the methods of quantum statistics, thermodynamic theories and numerical simulation, and also analyzed the effects of temperature, as well as the

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relativistic influence on the thermodynamic properties of the system [12]. We also researched relativistic thermodynamic properties of Fermi gas in a strong magnetic field at low temperatures, gave the corresponding analytical expression, and analyzed the influences of relativistic effect on the thermodynamic properties [13]. On this basis, using the methods of theoretical analysis and numerical simulation, we studied relativistic effect of the thermodynamic properties of Fermi gas in a strong magnetic field at both high and low temperatures, and then explored the influential mechanism of temperatures and the magnetic field on the relativistic effect.

2 The analytical expressions of the thermodynamic properties

According to Ref. [13], the relativistic part of the thermodynamic potential function of system can be shown as following

$$\Omega^R(\mu)=\frac{(\sigma B)^{5/2}}{8m^2c^2}\left(\frac{2m}{\pi}\right)^{5/2}\Gamma\left(\frac{5}{2}\right)\frac{TV}{\pi^2\hbar^3}\left\{-\frac{1}{\sqrt{2}}\log[1+e^{\frac{\mu}{T}}]\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}\right.$$
$$\left.+\frac{\sigma B}{\sqrt{2}\pi T}\frac{e^{\mu/T}}{1+e^{\mu/T}}\sum_{k=1}^{\infty}\frac{1}{k^{7/2}}-\sum_{k=1}^{\infty}\frac{\pi}{k^{5/2}}\frac{\sin(\pi k\mu/\sigma B-5\pi/4)}{sh(\pi^2kT/\sigma B)}\right\}, \tag{1}$$

and at low temperatures, the items of relativistic effect of energy, heat capacity, chemical potential can be expressed respectively as

$$U_1^R=-T^2\frac{\partial}{\partial T}\left[\frac{\Omega^R}{T}\right]_{V,N}=-\frac{3(\sigma B)^{5/2}\sqrt{m}}{8\pi^4\hbar^3c^2}V\left[\mu\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}-\frac{\sigma B}{\pi}\sum_{k=1}^{\infty}\frac{1}{k^{7/2}}\right.$$
$$\left.+\frac{\sqrt{2}\pi^2T^2}{\sigma B}\sum_{k=1}^{\infty}\frac{\sin\alpha\cos h\beta}{k^{3/2}\sin h^2\beta}\right], \tag{2}$$

$$C_1^R=\left[\frac{\partial U}{\partial T}\right]_{V,N}=-\frac{3(\sigma B)^{3/2}\sqrt{2m}}{4\pi\hbar^3c^2}TV\left[\sum_{k=1}^{\infty}\frac{\sin\alpha\cos h\beta}{k^{3/2}\sin h^2\beta}\right.$$
$$\left.+\frac{\pi^2T}{2\sigma B}\sum_{k=1}^{\infty}\frac{\sin\alpha(\sin h^2\beta-2\cos h^2\beta)}{k^{1/2}\sin h^3\beta}\right] \tag{3}$$

$$u_1^R=[\frac{\partial\Omega^R}{\partial N}]_{V,T}=-\frac{3(\sigma B)^{5/2}\sqrt{m}\varepsilon_F}{4\pi^4\hbar^3c^2n}V\left[\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}+\frac{\sqrt{2}\pi^2T}{\sigma B}\sum_{k=1}^{\infty}\frac{\cos\alpha}{k^{3/2}\sin h\beta}\right] \tag{4}$$

where $\alpha=\pi\mu k/\sigma B-\pi/4$, $\beta=\pi^2kT/\sigma B$.

Choosing $\mu=x\sigma B$, $T=y\sigma B$, then the expressions (2),(3)and (4) are written as

$$U_1^R=-\frac{3(\sigma B)^{7/2}\sqrt{m}}{8\pi^4\hbar^3c^2}V\left[x\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}-\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{1}{k^{7/2}}+\frac{\sqrt{2}\pi^3y^2}{\sigma B}\sum_{k=1}^{\infty}\frac{\sin(\pi kx-\pi/4)\cos h(\pi^2ky)}{k^{3/2}\sin h^2(\pi^2ky)}\right], \tag{5}$$