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Entangled trajectories based on Wigner function with negative values

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Abstract. Wigner function is a fundamental method to study the connection between quantum and classical system. Since negative value can be accepted by Wigner function even from a positive initial condition, they are various issues existing in corresponding interpretation as well as the development of numerical methods. We present the entangled trajectories based on the Wigner distributions with negative values.

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Key words: Entangled trajectory, Wigner function.

1 Introduction

Wigner function can reveal the deep insights between quantum system and classical world. It has the similar form compared with classical probability distribution which can be integrated over the whole phase space to obtain unit. However, negative values of Wigner function indicate that such a function can only be regarded as quasi-probability distribution. Such a characteristic cause many issues when we want to propagate Wigner function in a general form. Meanwhile, negative values, illustrated in Fig. 1, for instance, also make the distinctions between quantum and classical trajectories in the phase space. We choose a special case to propagate Wigner function precisely by eigenvalues and then analysis the natures of the entangled trajectories in detail.

2 Wigner function and Liouville equation

Here we only consider one dimensional case since multi-dimensional can be easily expanded. As a solution from time-dependent Schrödinger equation, $\Psi(x)$ can be used to

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construct Wigner function as follows:

$$\rho^w(q,p;t) = \frac{1}{2\pi\hbar}\int \Psi^*(q+x/2,t)\Psi(q-x/2,t)e^{(ipx/\hbar)}dx. \tag{1}$$

Combined with the time-dependent Schrödinger equation, we can obtain the quantum Liouville equation

$$\frac{\partial\rho^w}{\partial t} = -\frac{p}{m}\frac{\partial\rho^w}{\partial q} + \int J(q,\xi-p)\rho^w(q,\xi)d\xi, \tag{2}$$

where

$$J(q,\xi) = \frac{i}{2\pi\hbar^2}\int_{-\infty}^{\infty}[V(q+y/2)-V(q-y/2)]e^{(-iz\xi/\hbar)}dz. \tag{3}$$

Similar as a probability distribution, we can impose the continuity condition as

$$\frac{\partial\rho^w}{\partial t} = -\vec{\nabla}\cdot\vec{j}, \tag{4}$$

where $\vec{j} = (j_q, j_p)$ is the current vector in phase space and $\vec{\nabla} = (\partial/\partial q, \partial/\partial p)$is the gradient operator. The current is as follows

$$\vec{\nabla}\cdot\vec{j} = \frac{\partial}{\partial q}(\frac{p}{m}\rho^w) - \int_{-\infty}^{\infty} J(q,\xi-p)\rho^w(q,\xi,t)d\xi, \tag{5}$$

with

$$j_q = \frac{p}{m}\rho^w, \tag{6}$$

$$j_p = -\int_{-\infty}^{\infty}\Theta(q,\xi-p)\rho^w(q,\xi,t)d\xi, \tag{7}$$

and

$$\Theta(q,\eta) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}[V(q+y/2)-V(q-y/2)]\frac{e^{-i\eta y/\hbar}}{y}dy. \tag{8}$$

Since we can link the current density to density as well as velocity field in phase space by $\vec{j} = \rho^w\vec{v}$, we can finally deduce the equation of motion for the entangled trajectories as [1]

$$\begin{aligned} \dot{q} &= \frac{p}{m}, \\ \dot{p} &= \frac{1}{\rho^w}\int_{-\infty}^{\infty}\Theta(q,p-\xi)\rho^w(q,\xi)d\xi. \end{aligned} \tag{9}$$

We can use the above formula to realize the time evolution of the entangled trajectories.