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NEW ODE METHODS FOR EQUALITY CONSTRAINED OPTIMIZATION (1) — EQUATIONS*[1)]

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Abstract

To deal with equality constrained optimization problems (ECP), we introduce in this paper "(ECP)-equation", a class of new systems of ordinary differential equations for (ECP), containing a matrix parameter called (ECP)-direction matrix, which plays a central role in it, and a scalar parameter called (ECP)-rate factor. It is shown that by following the trajectory of the equation, a stationary point or hopefully a local solution can be located under very mild conditions. As examples, several schemes of (ECP)-direction matrices and (ECP)-rate factors are given to construct concrete forms of the (ECP)-equation, including almost all the existing projected gradient type versions as special cases. As will be shown in a subsequent paper where the implementation problems are considered in detail, application of an example of these forms results in encouraging performance in experiments.

§1. Introduction

Consider the following equality constrained optimization problem:

(ECP) $$\text{minimize (or maximize) } f(x), \tag{1.1}$$

$$\text{subject to } h(x) = 0. \tag{1.2}$$

It is assumed that both $f : D \subset R^n \to R$ and $h : D \subset R^n \to R^m$ are at least twice-continuously differentiable on domain D containing the feasible set X, the set of all x satisfying (1.2), and that

$$h(x) = [h_1(x), \cdots, h_m(x)]^T, \quad m < n, \tag{1.3}$$

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and the $n \times m$ transpose of its Jacobian

$$J(x) = [\nabla h_1(x), \cdots, \nabla h_m(x)] \tag{1.4}$$

has rank m at each $x \in X$. Note that X is a closed set since $h(x)$ is continuous on D.

ODE methods for the equality constrained optimization problem, methods which locate a solution of the problem by means of some system of ordinary differential equations, have been proposed by Arrow and Hurwicz[2], Fiaco and McCormick[15], Abadie and Corpentier[1], Evtushenko[13], Evtushenko and Zhadan[14], Tanabe[18–20], Botsaris[4–6], and others. A kind of prevalent viewpoint on ODE has been that the computational cost of ODE is higher than that of conventional ones such as SQP. However, recently Brown and Bartholomew-Biggs[11] have done their well-designed computational experiments, from which the conclusion drawn is right opposite: ODE methods for constrained optimization can perform very much better than some well-known and successful SQP methods. They therefore suggested: "ODE methods deserve more widespread attention than they have received so far from optimization researchers".

This work is done along this line. We propose a class of new ODE methods for problem (ECP), which can be regarded as extensions of ODE methods for unconstrained optimization problems developed by Pan[17]. In Section 2 we propose the (ECP)-equation, a class of systems of ordinary differential equations for (ECP) which contains a matrix parameter called (ECP)-direction matrix which plays a central role in the equation and a scalar parameter called (ECP)-rate factor. It is then shown that by following the trajectory defined by an (ECP)-equation, a stationary point or hopefully a local solution of (ECP) can be located under very mild conditions (the convergence rate will be discussed separately). In Section 3, some schemes of (ECP)-direction matrices and (ECP)-rate factors are suggested to construct concrete forms of the (ECP)-equation, including almost all the existing versions of projected gradient type as special cases. As will be shown in a subsequent paper where the implementation problems are considered in detail, application of an example of these forms results in performance much more superior to Tanabe's ODE version which is claimed to be more successful than some well-known SQP techniques by Brown and Bartholomew-Biggs[11].

First of all, for convenience, we will summarize, although by no means comprehensively, some terminology and results from the qualitative theory of ordinary differential equations that are relevant to our approaches. Almost all of them can be found, for example, in Hartman[16], Braun et al.[8], so most of the proofs related will not be given.