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A BOUNDARY ELEMENT APPROXIMATION OF A SIGNORINI PROBLEM WITH FRICTION OBEYING COULOMB LAW*

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Abstract

In this work, a Signorini problem with Coulomb friction in two dimensional elasticity is considered. Based on a new representation of the derivative of the double-layer potential, the original problem is reduced to a system of variational inequalities on the boundary of the given domain. The existence and uniqueess of this system are established for a small frictional coefficient. The boundary element approximation of this system is presented and an error estimate is given.

1. Introduction

The analysis of elastic bodies in contact which is a common problem for engineering design, has aroused the interest from both engineers and mathematicians. The most popular numerical techniques used to perform the analysis of contacting bodies are finite element method and boundary element method. In 1970, Wilson and Parson [1] used the finite element method to determine in a static analysis the surface contact stress between two plane bodies. Since then, finite element methods were applied widely: friction was introduced in the formulation through the Coulomb law [2,3], through various other frictional conditions and through the law of "non-local" and "nonlinear" type [4,5,6]. In the engineering literature, Anderson et al. [7] applied a direct boundary element method to two-dimensional frictionless static problems with successful results. Later, boundary element methods were extended rapidly for solving contact problems in elastostatics. For details, refer to [8,9,10].

Recently, a new approach of boundary reduction has been presented to reduce variational inequalities to corresponding boundary variational inequalities [11, 12]. In this paper, we extend this approach to a Signorini problem with friction obeying classical Coulomb law.

First, we reduce the original problem to a system of variational inequalities on the boundary based on a new representation of the derivative of the double-layer potential [13]. Then, we propose a boundary element method for the numerical approximation to this system and give the error estimates provided the frictional coefficient ν is not large.

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We begin by considering the following general Signorini problem with Coulomb friction. Let us assume that an elastic body is given in the natural configuration occupying a bounded domain $\Omega \subset \Re^2$, which is unilaterally supported by a rigid foundation. We look for a displacement field $u = (u_1, u_2)^T$ which is in the equilibrium state without body force:

$$\frac{\partial \sigma_{ij}(u)}{\partial x_j} = 0, \quad i = 1, 2, \quad \text{in } \Omega \tag{1.1}$$

where σ_{ij} denote the components of the stress tensor σ, and $\varepsilon = \{\varepsilon_{ij}\}$ denotes the strain tensor. The stress-strain relationships are given by the simplest Hook's law for homogeneous, isotropic materials:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}, \quad i, j = 1, 2,$$

where $\lambda \geq 0$, $\mu > 0$ are Lame's constants. Moreover,

$$\varepsilon_{ij} = \varepsilon_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i, j = 1, 2.$$

Suppose that the sufficiently smooth boundary Γ of Ω is composed of three nonempty parts Γ_D, Γ_F, Γ_C and $\bar{\Gamma}_F \cap \bar{\Gamma}_C = \emptyset$. On Γ_D, the elastic body Ω is supposed to be fixed, i.e.

$$u_i = 0, \quad \text{on } \Gamma_D, \quad i = 1, 2, \tag{1.2}$$

and is subjected to assigned surface force $g = (g_1, g_2)^T$ on Γ_F, i.e.

$$T(\partial_x, n(x))u = g, \quad \text{on } \Gamma_F, \tag{1.3}$$

where $n(x) = (n_1(x), n_2(x))^T$ is the unit outward normal to the boundary Γ, $T(\partial_x, n(x)) = T_{ij}(\partial_x, n(x))$ is defined by

$$T_{ij}(\partial_x, n(x)) = \lambda n_i(x) \frac{\partial}{\partial x_j} + \mu n_j(x) \frac{\partial}{\partial x_i} + \mu \delta_{ij} \frac{\partial}{\partial n(x)}, \quad i, j = 1, 2,$$

which is called the stress operator.

On Γ_C the body is unilaterally supported. This situation can be expressed by the classical unilateral boundary conditions of Signorini-Fichera type:

$$u_N \leq 0, \quad \sigma_N \leq 0, \quad u_N \sigma_N = 0, \quad \text{on } \Gamma_C. \tag{1.4}$$

Here $u_N = u \cdot n$ and $\sigma_N = \sigma_{ij} n_i n_j$ denote the normal components of displacements and stresses, respectively.