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FOURIER PSEUDOSPECTRAL-FINITE DIFFERENCE METHOD FOR INCOMPRESSIBLE FLOW*

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Abstract

A Fourier pseudospectral-finite difference scheme is proposed for unsteady Navier-Stokes equation. It is showed that the numerical solution keeps semi-discrete conservation. The strict error estimation is established. The numerical results are presented.

1. Introduction

In the past two decades, spectral and pseudospectral methods were widely used in fluid dynamics[1-3]. In particular, by using pseudospectral method it costs less computational time, and we can deal with non-linear terms easily. But pseudospectral method is not as stable as spectral method due to the aliasing interaction. To remedy this deficiency, we can use the filtering technique based on Bochner summation, given by Kuo Pen-yu[3-5] or the filtering technique proposed by Woodward, Collela and Vandeven[6-7].

In studying free-convection boundary layer on horizontal cylinder and some of other problems, we often meet semi-periodic problems[8-11]. There are two ways to solve such problems. The first is a combined method by using Fourier approximation with finite-difference or finite-element approach[9-19]. The second is Fourier-Chebyshev mixed approximation[20].

In this paper, we propose a Fourier pseudospectral-finite difference scheme for unsteady Navier-Stokes equation with periodic boundary condition in some directions. The orthogonality of trigonometric polynomials saves work. We approximate the continuity equation by the artificial compressibility given by Chorin, Lions and Guo Ben-yu[21-23]. Thus we can evaluate the speed and the pressure separately. Especially it is very difficult to construct the space of trial functions whose divergences vanish everywhere, for mixed Fourier pseudospectral-finite difference approximation. But the

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artificial compressibility helps us to avoid this hard job. In this paper, we also deal with the nonlinear terms by the skew symmetric decomposition, and so the numerical solution preserves the semi-discrete conservation. It also eliminates the effect of main nonlinear error terms in the analysis, and so we get better results. For increasing the stability of computation, we adopt the filtering technique of [3]. The numerical experiments show the advantages of this method.

The outline of this paper is as follows. We construct the scheme in Section 2 and present the numerical results in Section 3. We list some lemmas in Section 4. Then we prove the generalized stability and convergence in Section 5 and Section 6 respectively.

2. The Scheme

Let $x = (x_1, x_2, \cdots, x_n)^*, n = 2$ or 3. Assume that $n = n_1 + n_2$, where n_1 and n_2 are positive integers. For simplicity, let $x' = (x_1, \cdots, x_{n_1})^*$ and $x'' = (x_{n_1+1}, \cdots, x_n)^*$. Define $\Omega = \Omega_1 \times \Omega_2$, where

$$\Omega_1 = \{x'/0 < x_q < 1, 1 \le q \le n_1\}, \quad \Omega_2 = \{x''/0 \le x_q < 2\pi, n_1 + 1 \le q \le n\}.$$

Let $U(x,t)$ and $P(x,t)$ be the speed vector and the ratio of pressure over density respectively, $U = (U_1, \cdots, U_n)^*, \mu$ is the kinetic viscosity. We consider the following equation

$$\begin{cases} \frac{\partial U}{\partial t} + (U \cdot \nabla)U - \mu\nabla^2 U + \nabla P = f_1, & \text{in} \quad \Omega \times (0,T], \\ \nabla \cdot U = 0, & \text{in} \quad \Omega \times [0,T], \\ U(x,0) = U_0(x), & \text{in} \quad \overline{\Omega}, \\ P(x,0) = P_0(x), & \text{in} \quad \overline{\Omega}. \end{cases} \tag{2.1}$$

Suppose that the considered problem is partially periodic as in boundary layer problems (see [8-20]). It means that U, P and f_1 have the period 2π for $x_q, n_1 + 1 \le q \le n$. Furthermore, we suppose that the rest part of boundary is a non-slip wall and so the speed vanishes on $\partial\Omega_1$. In this case, the analysis of [24] also leads to $\frac{\partial P}{\partial n} = 0$, on $\partial\Omega_1$ approximately.

We are going to construct the scheme. Let $k = (k_{n_1+1}, \cdots, k_n), wherek_q$ are integers, and $|k| = (\sum_{q=n_1+1}^{n} k_q^2)^{\frac{1}{2}}$. For the pseudospectral approximation, let $\tilde{V}_N = Span\{e^{ik\cdot x''}/|k| \le N\}$. The real-valued subset of $\tilde{V}_N$ is denoted by V_N. Let P_N be the orthogonal projection from $[L^2(\Omega_1)]^n$ onto $(V_N)^n$ such that

$$\int_{\Omega_2} P_N v \cdot \overline{\omega} dx'' = \int_{\Omega_2} v \cdot \overline{\omega} dx'', \quad \forall \omega \in (V_N)^n.$$

Let $x''(j)$ be the interpolation nodes, i.e.,