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TWO DIMENSIONAL RIEMANN PROBLEM FOR GAS DYNAMICS SYSTEM IN THREE PIECES*[1)]

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Abstract

The Riemann problem for two-dimensional flow of polytropic gas with three constant initial data is considered. Under the assumption that each interface of initial data outside of the origin projects exactly one planar wave of shock, rarefaction wave or contact discontinuity, it is proved that only two kinds of combinations, **JRS** and **Js**, are reasonable. Numerical solutions are obtained by using a nonsplitting second order accurate MmB Scheme, and they efficiently reflect the complicated configurations and the geometric structure of solutions of gas dynamics system.

Key words: Two-dimensional Riemann problem, MmB scheme, gas dynamics.

1. Introduction

It is well known that the Riemann problem plays an essential role in developing one-dimensional theory of hyperbolic conservation laws[3] and it is the simplest one of general Cauchy problem and much easier to clarify the explicit structure of its solutions. On the other hand, the solution of the Cauchy problem can be locally approached by the solutions of Riemann problem. Hence the Riemann problem serves as the touchstone and the building block of mathematical theory of hyperbolic conservation laws. Of course, the most interesting and important model is the Euler equations in gas dynamics.

The Riemann Problem for two-dimensional unsteady flow of inviscid, polytropic gas with four piece constant in each quadrant was investigated by Zhang and Zheng in [10], and Chang, Chen and Yang in [2] etc.. With the characteristic analysis and the numerical method, a set of conjecture on the structure of solutions is formulated. Unfortunately, nothing analytic has eventually been solved, although there are still many mathematicians who present various simplified models and try to approach the conjecture and to explain the complicated configurations in gas dynamics system. Therefore it is worthwhile to consider much simpler Riemann initial data in two dimensions.

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The present paper deals in detail with the Riemann problem in three pieces for gas dynamics system, i.e.

$$\begin{cases} \rho_t + (\rho u)_x + (\rho v)_y = 0, \\ (\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0, \\ (\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0, \\ \Big(\rho\Big(e + \dfrac{u^2+v^2}{2}\Big)\Big)_t + \Big(\rho u\Big(h + \dfrac{u^2+v^2}{2}\Big)\Big)_x + \Big(\rho v\Big(h + \dfrac{u^2+v^2}{2}\Big)\Big)_y = 0, \end{cases} \tag{1.1}$$

where $\rho, (u,v)$ and p, $e = \dfrac{p}{(\gamma-1)\rho}$, $h = e + \dfrac{p}{\rho}$, $\gamma > 1$ denote density, velocity, pressure, specific internal energy, specific enthalty and polytropic index respectively. And Riemann data in three pieces are described as follows,

$$(\rho, p, u, v)|_{t=0} = T_i, \qquad (i = 1,2,3), \tag{1.2}$$

where T_i are constant states (See Fig.1.1), being selected under the assumption (H) that exactly one planar wave of shock, rarefaction wave or contact discontinuity issues from each interface of initial data outside of the origin. It's proved that only two cases, **JRS** and **three Js**, are in theory reasonable. Here we use a nonsplitting second order accurate MmB (locally Maximum-minimum Bounds preserving) scheme to obtain the numerical results for these two cases. MmB schemes are basically derived from the structure of the equation and the solution properties of scalar conservation laws[5], and are generalized to hyperbolic systems. The nonsplitting Mmb schemes have the second order accurate, high resolution and nonoscillatory properties, and have been used to solve many other problems concerning discontinuous solutions fruitfully[6,7].

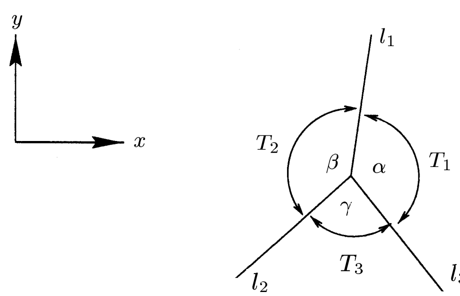

Fig.1.1 Distribution of the initial data

This paper is organized as follows. Section 2 gives the necessary preliminaries. In Section 3 we discuss the distribution of initial data carefully. And the characteristic analyses are presented and the corresponding numerical results are illustrated in Section 4.

2. Preliminaries

In this section we begin by recalling the main results in [2, 10] as our necessary preliminaries.

Noting that the dynamic similarity of (1.1) and lack of characteristic length parameter imply that the solutions be the functions of the variables ξ and η, where $\xi = x/t$, $\eta = y/t$, we seek the self-similar solutions.

$$(\rho, p, u, v) = (\rho, p, u, v)(\xi, \eta). \tag{2.1}$$