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A NEW STABILIZED FINITE ELEMENT METHOD FOR SOLVING TRANSIENT NAVIER-STOKES EQUATIONS WITH HIGH REYNOLDS NUMBER*

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Abstract

In this paper, we present a new stabilized finite element method for transient Navier-Stokes equations with high Reynolds number based on the projection of the velocity and pressure. We use Taylor-Hood elements and the equal order elements in space and second order difference in time to get the fully discrete scheme. The scheme is proven to possess the absolute stability and the optimal error estimates. Numerical experiments show that our method is effective for transient Navier-Stokes equations with high Reynolds number and the results are in good agreement with the value of subgrid-scale eddy viscosity methods, Petro-Galerkin finite element method and streamline diffusion method.

Mathematics subject classification: 65N30
Key words: Transient Navier-Stokes problems, High Reynolds number, The projection of the velocity and pressure, Taylor-Hood elements, The equal order elements.

1. Introduction

The description of incompressible flow problems is determined by the Navier-Stokes equations and lots of related studies to solve it have been reported (see, e.g., [2, 12, 15-17, 19, 28]). The stable and efficient mixed finite element approximation of the Navier-Stokes equations may face with two problems: violation of the discrete inf-sup condition and spurious oscillations due to the advection-dominated flows. Certainly, some finite element spaces satisfying the so-called inf-sup condition have been proposed in the past two decades, for instance, Taylor-Hood elements, Mini element. However, this condition prevents the most desirable choices of spaces to be adopted, such as the practical equal order elements especially the simplest and lowest $\mathbf{P}_1/P_1$ element. In order to circumvent the dilemma, an effective way is to take the stabilized finite element methods. Consistently stabilized methods (see, e.g., [11, 16, 35]) are developed using residuals of the momentum equation with the added stabilization terms, requiring approximation of the residual of second order derivations for Navier-Stokes equations, in which pressure and velocity derivatives vanish or are poorly approximated for low-order pairs. In order to overcome the drawback, many stabilized methods are proposed recently, of which the non-residual stabilized method has been a subject of active research. The pressure gradient projection method (see [2, 9]), the related local pressure gradient stabilization method (see [1, 6]) and the pressure projection stabilizations (see e.g., [5, 12, 18, 29, 30, 34]) are such examples.

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However, these stabilized methods only increase pressure stability and they will produce severely oscillating approximation solutions for Navier-Stokes equations with high Reynolds. The eddy viscosity model involving the non-residual stabilization has been proposed and attracted considerable attention. The large eddy viscosity simulation (LES) was proposed for advection-dominated flows which sought to simulate only the large scales of a turbulent flow accurately. In classical LES [21], the large scales are defined by an averaging in space which leads to serious problems if the flow is given in a bounded domain. A efficient remedy of the dilemma is the definition of the large scales in a different way, namely, by projection into appropriate spaces. This is the basis of variational multiscale method (VMM). VMM which the diffusion acts only on the finest resolved scales has been proposed by Hughes et al. (see [20]). John and Kaya ([25]) presented a subgrid-scale eddy viscosity methods (SEVM) for the convection-diffusion equation and gave a slightly more general interpretation of VMM. The consistently stabilized methods of Layton [26] and Guermond [14] fit into the framework as a generalized variational multiscale method. Recently, Kaya and John ([23, 24]), Zhang and He [36], and Chen et al. ([8]) extended the method to Navier-Stokes equations. In the above papers, the following stabilized term of SEVM was added to control the effect from the unresolved scales

$$M_1(\mathbf{u}_h, \mathbf{v}_h) = \alpha_1((I - \Pi_H^1)\nabla \mathbf{u}_h, (I - \Pi_H^1)\nabla \mathbf{v}_h), \tag{1.1}$$

where $H \geq h$.

SEVM is very effective in the convection dominated case and underresolved flow. However, the stabilized term in SEVM is the projection of the gradient of velocity and the projection has to be defined in piecewise constant space of different mesh size or in high order space when we choose the continuous piecewise linear element $\mathbf{P}_1$ to approximate the velocity. This causes much inconvenience to the numerical calculation. Moreover, these works (see [23, 24]) require velocity and pressure finite element spaces satisfying the inf-sup condition.

In our paper, we present a new stabilized method based on the projection of velocity and pressure for the transient Navier-Stokes equations with high Reynolds number. In the paper, the following stabilized term M is added to instead of the stabilized term M_1

$$M(\mathbf{u}_h, \mathbf{v}_h) = \alpha((I - \Pi_h)\mathbf{u}_h, (I - \Pi_h)\mathbf{v}_h). \tag{1.2}$$

In (1.1)–(1.2), α, α_1 are the stabilized parameters depending on the mesh size. Comparing with the stabilized term M_1, the projection of velocity in term M doesn't need to deal with the gradient of velocity and it is easy to be defined in the piecewise constant space for the continuous piecewise linear velocity space $\mathbf{P}_1$. The similar stabilization term M as our method has been used to solve the convection-dominated convection-diffusion equation in paper [7]. As is shown in [7], it is very effective in the convection dominated case. Besides, connections with the stabilized term M of the method in [7], artificial viscosity (AV) finite element methods, stream upwind Petrov Galerkin (SUPG) methods and VMM were presented. However, as far as we know, there are no researches on using this type of term to stabilize transient Navier-Stokes problems with high Reynolds number.

To get the fully discrete scheme, we use Taylor-Hood elements or the equal order elements in space and second order difference in time. The scheme is proven to possess the absolute stability and the optimal error estimates. Numerical experiments show that our method is effective for transient Navier-Stokes equations with high Reynolds number and has quite comparable numerical performances with SEVM in [24], Petrov-Galerkin finite element method and