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ENHANCED BLOCK-SPARSE SIGNAL RECOVERY PERFORMANCE VIA TRUNCATED ℓ_2/ℓ_{1-2} MINIMIZATION *

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Abstract

In this paper, we investigate truncated ℓ_2/ℓ_{1-2} minimization and its associated alternating direction method of multipliers (ADMM) algorithm for recovering the block sparse signals. Based on the block restricted isometry property (Block-RIP), a theoretical analysis is presented to guarantee the validity of proposed method. Our theoretical results not only show a less error upper bound, but also promote the former recovery condition of truncated ℓ_{1-2} method for sparse signal recovery. Besides, the algorithm has been compared with some state-of-the-art algorithms and numerical experiments have shown excellent performances on recovering the block sparse signals.

Mathematics subject classification: 68W40, 68P30, 94A08, 94A12
Key words: Compressed sensing, Block-sparse, Truncated ℓ_2/ℓ_{1-2} minimization method, ADMM.

1. Introduction

Compressed sensing (CS) [1–4] is a paradigm to acquire sparse, or compressible signals at a rate remarkably lower than that of the classical Nyquist sampling, which has attracted much attention in recent years. Nowadays, CS plays a very important role in many ways, for example, image processing [5], face recognition [6], subspace clustering [7] and other aspects. Recovering the sparse signals from linear measurements is an important subject of CS. The goal is to recover an unknown sparse signal $\mathbf{x} \in \mathbf{R}^{\mathbf{N}}$ from measurements $\mathbf{y} = A\mathbf{x} + \eta$, where $\mathbf{y} \in \mathbf{R}^{\mathbf{M}}$, $A \in R^{M\times N}(M \ll N)$ is a measurement matrix and $\eta \in \mathbf{R}^{\mathbf{M}}$ represents a vector of measurement errors. Compressed sensing solves the following constrained ℓ_0-minimization problem:

$$\min_{\mathbf{x}\in\mathbf{R}^{\mathbf{N}}} \|\mathbf{x}\|_{\mathbf{0}} \quad \mathbf{s.t.} \quad \mathbf{y} = \mathbf{A}\mathbf{x} + \eta, \tag{1.1}$$

where $\|\mathbf{x}\|_{\mathbf{0}}$ denotes the number of nonzero components of $\mathbf{x}$. However, the problem (1.1) is NP-hard [8]. In order to overcome this difficulty, ℓ_1 norm minimization was proposed as a

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substitution [2, 9, 10]:

$$\min_{\mathbf{x}\in\mathbf{R}^\mathbf{N}} \|\mathbf{x}\|_\mathbf{1} \quad \mathbf{s.t.} \quad \mathbf{y}=\mathbf{A}\mathbf{x}+\eta. \tag{1.2}$$

In the above formulation, the ℓ_1 norm is defined by $\|\mathbf{x}\|_\mathbf{1}=\sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{N}}|\mathbf{x_i}|$. The problem (1.2) can be solved by interior-point methods [11], alternating direction method of multipliers (ADMM) [12], iterative re-weighted least squares [13, 14, 36, 37] and so on. It is worth noting that Esser et al. proposed the ℓ_{1-2} minimization in [15]. This method was introduced as a sparsity penalty for nonnegative least squares problems and was later applied to sparse vector recovery. In [16], Yin et al. had proved that when the measurement matrix obeys some conditions related to RIP, the ℓ_{1-2} minimization method can exactly recover any sparse signals. In addition, their experimental results also showed that this method has better performance than the other methods when the matrix A is highly coherent. This optimization problem can be formulated as

$$\min_{\mathbf{x}\in\mathbf{R}^\mathbf{N}} \|\mathbf{x}\|_\mathbf{1}-\|\mathbf{x}\|_\mathbf{2} \quad \mathbf{s.t.} \quad \mathbf{y}=\mathbf{A}\mathbf{x}+\eta. \tag{1.3}$$

It is known that the assumption on the sparsity of signals heavily affects the recovery performance of CS. In dealing with sparse signal recovery problems, the traditional compressed sensing methods may ignore their deeper sparse structures, such as block sparsity. Improving the original ℓ_0 norm minimization, and letting it reflect the relevant characteristics of the block has become a natural choice. In this paper, we address this problem that the signal is block sparse. Such structured sparse signals extensively emerge in various applications. Prominent examples include machine learning [17], channel estimation [18] and source location [19]. Moreover, potential applications and recovery algorithms can be found in a series of recent references (e.g. [20–24]). To facilitate the description of the block sparse signal, we assume that there are m blocks with block size $d=N/m$ in $\mathbf{x}$. That is to say, we can write the signal $\mathbf{x}$ as

$$\mathbf{x}=[\underbrace{x_1,\cdots,x_d}_{\mathbf{x[1]}},\underbrace{x_{d+1},\cdots,x_{2d}}_{\mathbf{x[2]}},\cdots,\underbrace{x_{N-d+1},\cdots,x_N}_{\mathbf{x[m]}}]^\mathbf{T},$$

where $\mathbf{x}[\mathbf{i}]$ denotes the ith block of $\mathbf{x}$. We call a vector $\mathbf{x}$ block k-sparse if it has at most k nonzero blocks, i.e., $\|\mathbf{x}\|_{\mathbf{2},\mathbf{0}}\le\mathbf{k}$, where $\|\mathbf{x}\|_{\mathbf{2},\mathbf{0}}=\sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{m}}\mathcal{I}(\|\mathbf{x}[\mathbf{i}]\|_\mathbf{2})$ and $\mathcal{I}(x)$ represents the indicator function. The block sparse recovery is to deal with the following problem

$$\min_{\mathbf{x}\in\mathbf{R}^\mathbf{N}} \|\mathbf{x}\|_{\mathbf{2},\mathbf{0}} \quad \mathbf{s.t.} \quad \mathbf{y}=\mathbf{A}\mathbf{x}+\eta. \tag{1.4}$$

Similar to non-convex model (1.1), solving (1.4) is also intractable. To recover block sparse signal in a more tractable way, Lin and Li [25] and Wang et al. [26] proposed the mixed ℓ_2/ℓ_1 norm minimization method:

$$\min_{\mathbf{x}\in\mathbf{R}^\mathbf{N}} \|\mathbf{x}\|_{\mathbf{2},\mathbf{1}} \quad \mathbf{s.t.} \quad \mathbf{y}=\mathbf{A}\mathbf{x}+\eta, \tag{1.5}$$

where $\|\mathbf{x}\|_{\mathbf{2},\mathbf{1}}=\sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{m}}\|\mathbf{x}[\mathbf{i}]\|_\mathbf{2}$. This method was posed to recover block sparse signals via using the ℓ_2 and ℓ_1 norms simultaneously. Specially, the ℓ_1 norm characterizes the inter-block sparsity in $[\|\mathbf{x}[\mathbf{1}]\|_\mathbf{2},...,\|\mathbf{x}[\mathbf{m}]\|_\mathbf{2}]^\mathbf{T}$, the ℓ_2 norm characterizes the intra-block cooperation in $\mathbf{x}[\mathbf{i}]$. Recently, Wang et al. [27] introduced the ℓ_2/ℓ_{1-2} minimization form:

$$\min_{\mathbf{x}\in\mathbf{R}^\mathbf{N}} \|\mathbf{x}\|_{\mathbf{2},\mathbf{1}}-\|\mathbf{x}\|_\mathbf{2} \quad \mathbf{s.t.} \quad \mathbf{y}=\mathbf{A}\mathbf{x}+\eta, \tag{1.6}$$