Journal of Computational Mathematics
Vol.39, No.2, 2021, 192–206.
http://www.global-sci.org/jcm
doi:10.4208/jcm.1910-m2019-0078

STABILITY ANALYSIS OF THE SPLIT-STEP THETA METHOD FOR NONLINEAR REGIME-SWITCHING JUMP SYSTEMS*

Guangjie Li
School of Mathematics and Statistics, Guangdong University of Foreign Studies, Guangzhou 510420, China
Email: scutliguangjie@163.com
Qigui Yang[1)]
Department of Mathematics, South China University of Technology, Guangzhou 510640, China
Email: qgyang@scut.edu.cn

Abstract

In this paper, we investigate the stability of the split-step theta (SST) method for a class of nonlinear regime-switching jump systems–neutral stochastic delay differential equations (NSDDEs) with Markov switching and jumps. As we know, there are few results on the stability of numerical solutions for NSDDEs with Markov switching and jumps. The purpose of this paper is to enrich conclusions in such respect. It first devotes to show that the trivial solution of the NSDDE with Markov switching and jumps is exponentially mean square stable and asymptotically mean square stable under some suitable conditions. If the drift coefficient also satisfies the linear growth condition, it then proves that the SST method applied to the NSDDE with Markov switching and jumps shares the same conclusions with the exact solution. Moreover, a numerical example is demonstrated to illustrate the obtained results.

Mathematics subject classification: 60H10, 60H35, 34k34, 65L20.
Key words: Exponential mean-square stability, Neutral stochastic delay differential equations, Split-step theta method, Markov switching and jumps.

1. Introduction

Stochastic dynamic systems which are often described by stochastic differential equations (SDEs) have come to play an important role in many branches of science and industry, such as economic, finance, biology, medicine and so on. However, since most SDEs are nonlinear and cannot be solved explicitly, hence numerical approximations have become an important tool in the study of SDEs. A lot of numerical results for the SDEs have been obtained [1–6]. In practice, many stochastic dynamic systems not only depend on the present and past states, but also depend on the state derivatives. NSDDEs are often used to described such systems, and numerical results for NSDDEs have received a lot of attention [7–10]. Especially, Liu and Zhu [11] proved that the split-step theta method with $\theta \in [0,1]$ applied to autonomous NSDDEs is exponentially mean square stable. Zong et al. [12] showed that the exponential mean square stability of the split-step theta method for NSDDEs. Authors in [12] proved that if the exact solution is exponentially mean square stable, then the corresponding split-step theta method is also exponentially mean square stable with $\theta \in [0, 1/2]$ under some conditions,

* Received March 25, 2019 / Revised version received August 14, 2019 / Accepted October 24, 2019 / Published online December 17, 2019 /
1) Corresponding author

but for $\theta \in (1/2, 1]$, the split-step theta method can reproduce the exponential mean square stability unconditionally.

On the other hand, many practical situations where they may experience abrupt changes in their structure and parameters, a class of systems with the continuous-time Markov chain, which are the so-called regime-switching systems, have been used to describe them. Recently, stochastic regime-switching systems, including SDEs with Markov switching (SDEwMSs), stochastic delay differential equations with Markov switching (SDDEwMSs), NSDDEs with Markov switching (NSDDEwMSs) have attracted a great deal of attention (see [13–16] and the references therein).

As we know, a Brownian motion is a continuous stochastic process. However, many real systems may suffer from jump type stochastic abrupt perturbations, such as stochastic failures, earthquakes, hurricanes, and so on. In these cases, we can not use Brownian motions to describe these systems. Therefore, it may be reasonable to use jump processes to cope with such jump type discontinuous systems [17–20]. Recently, many results on NSDDEs with jumps have appeared [21–24]. Among of these literature, Palanisamy and Chinnathambi [24] obtained some new sufficient conditions for the approximate controllability of a class of second-order NSDDEs with infinite delay and Poisson jump. Mo et al. [22,23] investigated the exponential stability of the theta method and split-step theta method for NSDDEs with jumps, respectively. It is a pity that the above mentioned work did not take the Markov switching into consideration. In order to achieve the goal in the present paper, we need to overcome the difficulties of coping with the Markov switching and jumps, therefore, more techniques are needed to address NSDDEs with Markov switching and jumps.

So far, it is also well known that there are few results on the stability analysis of the split-step theta method for NSDDEs with Markov switching and jumps. The purpose of this paper is to establish some new results on the exponential mean-square stability and asymptotical mean-square stability of the split-step theta method for NSDDEs with Markov switching and jumps. Moreover, the almost sure exponential stability of the trivial solution and split-step theta numerical solutions can also be obtained.

The remainder of this paper is organized as follows. In Section 2, it introduces some preliminaries. In Section 3, it devotes to show that the trivial solution is exponentially mean-square stable and asymptotically mean-square stable. In Section 4, we are in a position to investigate the exponential mean-square stability and the asymptotical mean-square stability of the split-step theta numerical solution. In Section 5, to show the effectiveness of the obtained theory, an illustrative example is provided.

2. Preliminary

Throughout this paper, unless otherwise specified, we use the following notations. R^n denotes the n-dimensional Euclidean space, and $|x|$ denotes the Euclidean norm of a vector x. Let $R = (-\infty, +\infty)$, $R^+ = [0, +\infty)$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. its right continuous and $\mathcal{F}_0$ contains all P-null sets). Let $\tau > 0$ and $C([-\tau, 0]; R^n)$ denote the family of all continuous R^n-valued functions φ defined on $[-\tau, 0]$. Let $C^b_{\mathcal{F}_0}(\Omega; R^n)$ be the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau, 0]; R^n)$ with the norm $\|\varphi\| = \sup_{-\tau\leq\theta\leq 0} |\varphi(\theta)|$. $\langle x, y\rangle$ or $x^T y$ represents the inner product of $\forall x, y \in R^n$. Let $\{r(t)\}_{t\geq 0}$ be a right-continuous Markov chain on the complete probability space taking values in a finite state space $S = \{1, \cdots, N\}$ with the generator $\Gamma = (\gamma_{ij})_{N\times N}$