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LOCAL GAUSSIAN-COLLOCATION SCHEME TO APPROXIMATE THE SOLUTION OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS USING VOLTERRA INTEGRAL EQUATIONS*

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Abstract

This work describes an accurate and effective method for numerically solving a class of nonlinear fractional differential equations. To start the method, we equivalently convert these types of differential equations to nonlinear fractional Volterra integral equations of the second kind by integrating from both sides of them. Afterward, the solution of the mentioned Volterra integral equations can be estimated using the collocation method based on locally supported Gaussian functions. The local Gaussian-collocation scheme estimates the unknown function utilizing a small set of data instead of all points in the solution domain, so the proposed method uses much less computer memory and volume computing in comparison with global cases. We apply the composite non-uniform Gauss-Legendre quadrature formula to estimate singular-fractional integrals in the method. Because of the fact that the proposed scheme requires no cell structures on the domain, it is a meshless method. Furthermore, we obtain the error analysis of the proposed method and demonstrate that the convergence rate of the approach is arbitrarily high. Illustrative examples clearly show the reliability and efficiency of the new technique and confirm the theoretical error estimates.

Mathematics subject classification: 34A08, 45D05, 65G99, 65L60.
Key words: Nonlinear fractional differential equation, Volterra integral equation, Gaussian-collocation method, Meshless method, Error analysis.

1. Introduction

Many problems of mathematical physics, engineering and biology can be stated in the form of differential equations including non-integer order derivatives which are an important subdivision of fractional calculus [10,12]. The analytic results on existence, uniqueness and continuous upon data to the solution of fractional differential equations have been studied by many authors [33, 36]. Finding the approximate solution for these types of differential equations is one of the most significant problems in computational mathematics. Homotype perturbation method [1], Haar wavelets method [13], Adomian decomposition method [14], extrapolation

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method [16], predictor-corrector method [15], fractional linear multi-steps method [25], generalized differential transform method [30], variational iteration method [38], finite difference method [40], hybrid function method [27], multiquadric radial basis function [39], explicit methods [20], and Adams method [2] have been applied to solve these types of differential equations.

The first appearance of Gaussians as a type of radial basis functions (RBFs) seems to be in the implicit reconstruction of a surface from unorganized points known under the name blobby surfaces or metaballs [11]. Riemenschneider and Sivakumar [34] have studied Gaussians for interpolating a function over a set of scattered points and their convergence behavior for varying parameters in the exponential in 1999. Since the classical Gaussians are global functions, the resultant coefficient matrix with respect to them will be ill-conditioned when many points in the domain are considered for obtaining high-order accurate results [37]. To overcome these difficulties, the Gaussians with local supports have been introduced in the manuscripts [22,35] so-called local Gaussians. Since local Gaussians use the only geometrical data fallen within local influence domain, they require much less computational work in comparison with the globally types. Another additional benefit of this formulation is that we have much more freedom in choosing the shape parameters [35].

This paper presents a computational scheme to solve the following nonlinear fractional differential equation:

$$D_a^{\alpha}u(x) + AD_a^{\beta}u(x) = f(x, u(x)), \quad a \leq x \leq b, \tag{1.1}$$

with initial conditions

$$u(a) = u_0, \quad u'(a) = u_1, \quad u^{(k)}(a) = 0, \quad k = 2, \ldots, n-1, \tag{1.2}$$

where $\alpha, \beta > 0,\ \alpha, \beta \in \mathbb{Q},\ \alpha \geq \beta$, the unknown function $u(x)$ must be determined, the given function f is nonlinear respect to the variable u and $A \in \mathbb{R}$ is a non-zero constant. The method first reduces the solution of the fractional differential equation (1.1) to the solution of a fractional Volterra integral equation. This conversion causes the initial conditions to be occulted in the integral equation and so the scheme requires no strategies to justify these conditions over the approximate solution. On the other hand, the proposed method does not apply any numerical evaluations for fractional derivatives, unlike most available methods. To handle the product integration methods as well-known schemes for solving fractional integral equations, we need a uniform or non-uniform mesh on the solution domain [9]. The method proposed in the current paper utilizes the locally supported Gaussians as basis in the collocation which approximate a the solution of the equivalent integral equation without any mesh generations. Since the fractional integrals appeared in the method cannot be computed by the classical integration schemes, we introduce a particular integration rule based on the use of the composite Gauss-Legendre quadrature formula. The error analysis of the proposed method is provided. The convergence accuracy of the new technique is examined over several illustrated examples.

In the following, we consider some advantages of the proposed method which make it attractive compared with other schemes for solving fractional differential equations:

- The method does not require any domain elements, so it is identified as a meshless method.
- Since the technique applies only geometrical nodal points fallen within local influence domain, it obtains more accurate results using much fewer volume computing.
- The algorithm of the method can be simply implemented on computers and is more flexible for most classes of fractional differential equations.