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SCHWARZ METHOD FOR FINANCIAL ENGINEERING*

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Abstract

Schwarz method is put forward to solve second order backward stochastic differential equations (2BSDEs) in this work. We will analyze uniqueness, convergence, stability and optimality of the proposed method. Moreover, several simulation results are presented to demonstrate the effectiveness; several applications of the 2BSDEs are investigated. It is concluded from these results that the proposed the method is powerful to calculate the 2BSDEs listing from the financial engineering.

Mathematics subject classification: 65C30, 65C05, 60H35, 60H07, 60J75.
Key words: 2BSDE, Schwarz method, Domain decomposition, Viscosity solution, Stochastic volatility models.

1. Introduction

The search for fast and efficient schemes of the backward stochastic differential equations (BSDEs) is a challenging task. The study of parallel and distributed solutions is thus important. These solutions employ two basic forms: domain decomposition and function decomposition (task decomposition). The former is based on original BSDEs. It yields several sub-systems in parallel, on each subdomain. The latter sub-divides the system of BSDEs into many components (or task) to be parallelized.

Motivated by applications and probabilistic numerical methods for second order BSDEs (2BSDEs), Cheridito et al. (2007) considered the connection between the 2BSDEs and fully nonlinear parabolic PDEs. This connection is found through the dependence of a drift part. In addition, Soner et al. (2012) proposed a form of 2BSDEs in connection with G-expectations and G-martingales. We now present our discussed 2BSDEs, written by

$$\begin{cases} dY_t = -f(t, X_t, Y_t, Z_t, \Gamma_t)dt + Z_t dB_t, \\ dZ_t = -A_t dt - \Gamma_t dB_t, \quad t \in [0, T], \end{cases} \tag{1.1}$$

where $Y_T = \phi(X_T)$ and $Z_T = z$. A_t and Γ are all measurable processes. ϕ and f are all deterministic functions. X_t is a diffusion process. $B_t = (B_t^1, \cdots, B_t^r)^T$ is a r-dimensional Brownian motion. Here $f(t, X_t, Y_t, Z_t, \Gamma_t) = f(t, X_t, Y_t, Z_t) + Tr(\Gamma_t)/2$. It is worthy noting

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that, the system of 2BSDEs (1.1) is a special case of G-BSDEs mentioned by Hu et al. (2014). Based on stochastic integral theory, the 2BSDEs can be listed as

$$Y_t = \phi(X_T) + \int_t^T f(s, X_s, Y_s, Z_s, \Gamma_s)ds - \int_t^T Z_s dB_s, \tag{1.2a}$$

$$Z_t = z + \int_t^T A_s ds + \int_t^T \Gamma_s dB_s. \tag{1.2b}$$

With $|\pi| = \max_i |t_{i+1} - t_i|$, and the partition $\pi = \{0 = t_0 \leq \cdots \leq t_i \leq \cdots \leq t_N = T\}$ on $[0,T]$. $Y_T^\pi = \phi(X_T^\pi)$. X^π is a corresponding discretisation of X. We then have the following time Euler discretisation of the 2BSDEs:

$$\begin{cases} Y_{t_i}^\pi = E_i^\pi[Y_{t_{i+1}}^\pi] + f(t_i, X_{t_i}^\pi, Y_{t_i}^\pi, Z_{t_i}^\pi, \Gamma_{t_i}^\pi)(t_{i+1} - t_i), \\ Z_{t_i}^\pi = \dfrac{1}{(t_{i+1} - t_i)} E_i^\pi[Y_{t_{i+1}}^\pi (B_{t_{i+1}} - B_{t_i})], \\ \Gamma_{t_i}^\pi = \dfrac{1}{(t_{i+1} - t_i)} E_i^\pi[Z_{t_{i+1}}^\pi (B_{t_{i+1}} - B_{t_i})], \\ A_{t_i}^\pi = \dfrac{1}{(t_{i+1} - t_i)} E_i^\pi[Z_{t_{i+1}}^\pi]; i \in [0, N-1], \end{cases} \tag{1.3}$$

which is similar to the works of Cheridito et al. (2007) and Soner et al. (2012). Under several regularity conditions, a solution exists on $\hat{Y}_t = u(t, X_t)$. We consider the following backward second order parabolic PDEs on $[0,T) \times \mathbb{R}^r$, given by

$$\partial_t u(t,x) + f(t, x, u(t,x), Du(t,x), D^2 u(t,x)) = 0, \tag{1.4a}$$

$$u(T,x) = \phi(x), \tag{1.4b}$$

where $x = (x^1, \cdots, x^r) \in \mathbb{R}^r$. Here

$$\partial_t = \frac{\partial}{\partial t}, \quad Du = (D_i u), \quad D^2 u = (D_{ij} u), \quad D_{ij} = D_i D_j, \quad D_i = \frac{\partial}{\partial x^i}.$$

Then

$$\hat{Y}_t = u(t, X_t), \quad \hat{Z}_t = Du(t, X_t), \quad \hat{\Gamma}_t = D^2 u(t, X_t), \quad \hat{A}_t = \mathcal{L} Du(t, X_t)$$

is a solution of the 2BSDEs. The Dynkin operator $\mathcal{L}$ of X is without the drift term, see also Cheridito et al. (2007).

As to the aforementioned 2BSDEs, we give several conditions useful for the uniqueness of solution.

(A0). For $(t, x, y, z) \in [0,T] \times \mathbb{R}^r \times \mathbb{R} \times \mathbb{R}^d$, $f(t, x, y, z, \gamma) \geq f(t, x, y, z, \tilde{\gamma})$ whenever $\gamma \leq \tilde{\gamma}$ with $\gamma, \tilde{\gamma} \in \mathbb{R}^d$. For all $t \in [0,T]$, the PDEs (1.4) satisfy the comparison principle.

(A1). f satisfies Lipschitz condition when $\|v\|_2 + \|N_0\|_2 + \|Q\|_2$ is bounded. That is, for fixed t, there exists a constant K_0 (> 0) satisfying

$$\Big\| f(t, x, u + v, P + Q, M + N_0) - f(t, x, u, P, M) \Big\|_2 \leq \frac{K_0}{T - t} \Big(\|v\|_2 + \|N_0\|_2 + \|Q\|_2 \Big).$$

(A0) constraints the viscosity of Equations (1.4). (A1) constraints the elements of $f(t, x, \cdot, \cdot, \cdot)$, and is a stronger consistency condition. In this case, Equations (1.4) have a solution. A uniqueness theorem and a simplified proof are respectively as follows.