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A CELL-CENTERED ALE METHOD WITH HLLC-2D RIEMANN SOLVER IN 2D CYLINDRICAL GEOMETRY*

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Abstract

This paper presents a second-order direct arbitrary Lagrangian Eulerian (ALE) method for compressible flow in two-dimensional cylindrical geometry. This algorithm has half-face fluxes and a nodal velocity solver, which can ensure the compatibility between edge fluxes and the nodal flow intrinsically. In two-dimensional cylindrical geometry, the control volume scheme and the area-weighted scheme are used respectively, which are distinguished by the discretizations for the source term in the momentum equation. The two-dimensional second-order extensions of these schemes are constructed by employing the monotone upwind scheme of conservation law (MUSCL) on unstructured meshes. Numerical results are provided to assess the robustness and accuracy of these new schemes.

Mathematics subject classification: 76M12, 35L65, 65M06.
Key words: Riemann solver, ALE, HLLC-2D, Cylindrical geometry.

1. Introduction

In some computational fluid dynamics (CFD) applications such as inertial confinement fusion (ICF), the physical models have multi-material interfaces and moving boundaries. Lagrangian methods can track material interfaces and capture contact discontinuity sharply in multi-material fluid flows, but they suffer the large scale mesh deformations in the case of complicated flows. The arbitrary Lagrangian Eulerian (ALE) methods are powerful tools to solve these problems on the moving mesh framework [15]. Our goal is to propose a new robust ALE cell-centered scheme for the compressible flow in two-dimensional cylindrical geometry. Let us briefly give an overview of the cell-centered Lagrangian or ALE schemes before introducing our schemes.

The cell-centered Lagrangian method was originally introduced by Godunov [12], whose primary variables (density, velocity and pressure) are defined on the center of meshes. The numerical flux across an edge of mesh is computed by a one-dimensional exact or approximate Riemann solver, and the node velocity needs to be calculated by additional algorithms, such as

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the least square procedure [10] or some algebra conditions [14]. Nevertheless, such mesh moving approaches lead to some numerical instabilities when computing strong shock problems [26]. In fact, the fluxes aren't computed in a coherent manner with the nodal velocity in the early Lagrangian CAVEAT algorithm [10]. In order to keep the compatibility between the fluxes and nodal velocity, Després and Mazeran [9], and Maire et al. [16] designed an approximate Riemann solver located at vertex. The modified calculation results are quite robust. From then on, a series of contributions to the development of this method is performed. Burton et al. extended to solid dynamics by proposing a new nodal Riemann-like method [3]. Carré et al. extended Després and Mazeran's method to an arbitrary given dimension [5]. Maire et al. developed high-order Lagrangian schemes on two-dimensional Cartesian geometry [17] and cylindrical geometry [18]. Shen et al. constructed a direct ALE algorithm on Cartesian geometry [25, 26]. For the development of cell-centered ALE method in cylindrical geometry, Loubére et al. introduced a reconnection algorithm (ReALE) to change the topology of meshes [19] and Friess et al. presented an interface reconstruction, Moment Of Fluid (MOF), for the numerical simulation of multi-material compressible fluid flows [20].

Two-dimensional numerical schemes in cylindrical geometry are very important for fluid hydrodynamic problems. There are three discrete methods in cylindrical coordinate: the control volume method, the area-weighted method and the plane scheme with an additional geometric source term [2, 29]. The plane scheme with a geometric source term has shown that the shock position is wrong in some typical tests such as the spherical Sedov problem [27]. Therefore, we only focus on the former two methods in this paper. The control volume scheme conservers momentum, but doesn't preserve spherical symmetry on equal-angle-zoned grids [7, 18]. On the other hand, the area-weighted scheme [4, 18, 21], which preserves spherical symmetry exactly on equal-angle-zoned grids, doesn't maintain momentum conservation. It is reasonable to worry about the potential trouble caused by the non-conservation property of the area-weighted scheme [7,8]. But as far as we know, almost all numerical tests show the area-weighted method don't bring larger errors than the control volume scheme. Thus the area-weighted scheme is still one of the best choices to simulate cylindrical problems [27].

This paper presents a second-order cell-centered direct ALE scheme for compressible flow in cylindrical geometry. Our ALE method introduces two half-face fluxes on mesh edges and ensures the compatibility between the edge flux and the nodal velocity, which is different from traditional cell-centered Lagrangian methods. Based on the HLLC approximate Riemann solver, we construct a genuine two dimensional solver HLLC-2D in 2D cylindrical geometry. An important fact is that when the nodal moving velocity or the fluid velocity is zero, our ALE scheme degenerates to the Eulerian and Lagrangian method naturally. The new algorithm can be viewed as a cylindrical extension from the Cartesian coordinate in the ALE framework [26]. Moreover, we define a control volume scheme and an area-weighted scheme in the cylindrical geometry, and extend these methods to second-order by the classical monotone upwind scheme of conservation law (MUSCL) algorithm on unstructured meshes [30]. For the spherical Noh problem, our area-weighted scheme can preserve better spherical symmetry on Cartesian grids, which has no oscillation on the density plateau behind the shock wave.

The outline of this paper is as follows: In Section 2 the governing equations and discrete notations of the gas hydrodynamics are written in the cylindrical geometry. In Section 3, a new Riemann solver in 2D cylindrical geometry with ALE framework is described, including the nodal solver and numerical fluxes. Two different discrete formulations, a control volume scheme and an area-weighted scheme, have been constructed and our MUSCL-type extension