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MODIFIED ALTERNATING POSITIVE SEMIDEFINITE SPLITTING PRECONDITIONER FOR TIME-HARMONIC EDDY CURRENT MODELS*

Yifen Ke and Changfeng Ma[1)]
College of Mathematics and Informatics & FJKLMAA, Fujian Normal University, Fuzhou 350117, China
Email: keyifen@fjnu.edu.cn, macf@fjnu.edu.cn

Abstract

In this paper, we consider a modified alternating positive semidefinite splitting preconditioner for solving the saddle point problems arising from the finite element discretization of the hybrid formulation of the time-harmonic eddy current model. The eigenvalue distribution and an upper bound of the degree of the minimal polynomial of the preconditioned matrix are studied for both simple and general topology. Numerical results demonstrate the effectiveness of the proposed preconditioner when it is used to accelerate the convergence rate of Krylov subspace methods such as GMRES.

Mathematics subject classification: 65F08, 65F10, 65N22.
Key words: Time-harmonic eddy current model; Saddle point problem; Eigenvalue distribution; Preconditioner.

1. Introduction

The time-harmonic eddy current model is often used to simulate the electromagnetic phenomena concerning alternating currents at low frequencies (see [15,42–44]). The main equations of this model are Faraday's law

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \mathbf{J}_e \quad \text{in } \Omega, \tag{1.1}$$

and Ampère's law

$$\nabla \times \mathbf{E} = -\mathrm{i}\omega\mu \mathbf{H} \quad \text{in } \Omega, \tag{1.2}$$

where $\mathbf{E}$, $\mathbf{H}$ and $\mathbf{J}_e$ are the electric field, the magnetic field and a given generator current, respectively. Here, $\nabla\times$ is the curl operator, i.e.,

$$\begin{aligned}\nabla \times \mathbf{v} :=& \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} \\ =& \Big(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\Big)\mathrm{i} + \Big(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\Big)\mathrm{j} + \Big(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\Big)\mathrm{k},\end{aligned}$$

where $\mathbf{v} = (v_x, v_y, v_z)^{\mathrm{T}}$ is a vector valued function.

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1) Corresponding author.

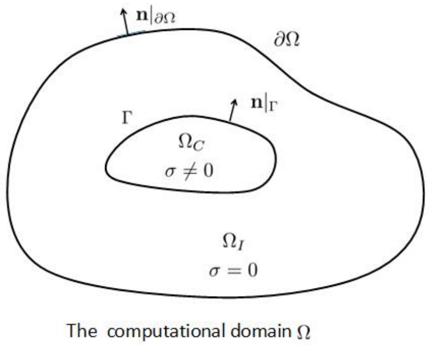

Fig. 1.1. The classical computational domain of the time-harmonic eddy current models.

Fig. 1.1 presents a classical computational domain Ω of the time-harmonic eddy current models. Without loss of generality, the computational domain $\Omega \subset \mathbb{R}^3$ is assumed to be a simply connected Lipschitz polyhedron, which consists of a conducting region $\Omega_C \subset \Omega$ and its complement $\Omega_I = \Omega \setminus \overline{\Omega}_C$, with $\overline{\Omega}_C$ and $\overline{\Omega}_I$ representing the closed sub-domains corresponding to Ω_C and Ω_I, respectively. We assume that Ω_C and Ω_I are Lipschitz polyhedrons and that Ω_C is connected but not necessarily simply connected. The magnetic permeability μ is assumed to be a symmetric and uniformly positive-definite 3×3 tensor with entries in $\mathbb{L}^\infty(\Omega)$. The same assumption holds for the electric conductivity σ in the conducting region whereas it is null in nonconducting regions. The real scalar constant $\omega \neq 0$ is a given angular frequency. In addition, the symbol i denotes the imaginary unit, i.e., $\mathrm{i} = \sqrt{-1}$, $\partial\Omega$ denotes the boundary of the domain Ω, $\Gamma = \overline{\Omega}_C \cap \overline{\Omega}_I$, and $\mathbf{n}_{|\partial\Omega}$ and $\mathbf{n}_{|\Gamma}$ represent the unit outward normal vectors on Ω and on Γ pointing toward Ω_I, respectively. For a given vector field $\mathbf{v}$ defined in Ω, we denote by $\mathbf{v}_L$ the restriction to Ω_L $(L = C, I)$.

Since $\sigma \equiv 0$ in the nonconducting region, the generator current has to satisfy the compatibility conditions

$$\nabla \cdot \mathbf{J}_{e,I} = 0 \qquad \text{in } \Omega_I, \tag{1.3}$$

$$\int_\Gamma \mathbf{J}_{e,I} \cdot \mathbf{n}|_\Gamma \mathrm{d}S = 0, \tag{1.4}$$

where $\mathrm{d}S$ is a surface increment. Here, $\nabla\cdot$ is the divergence operator, i.e.,

$$\nabla \cdot \mathbf{v} := \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z},$$

where $\mathbf{v} = (v_x, v_y, v_z)^{\mathrm{T}}$ is a vector valued function.

The equations (1.1) and (1.2) do not completely determine the electric field in Ω_I, and it is necessary to require the gauge condition

$$\nabla \cdot (\epsilon \mathbf{E}_I) = 0 \quad \text{in } \Omega_I, \tag{1.5}$$

where ϵ is the dielectric permittivity, which is also assumed to be a symmetric uniformly positive definite tensor with entries in $\mathbb{L}^\infty(\Omega)$.

Most often, some suitable boundary conditions must be assigned in the boundary of the computational domain Ω, for example, the tangential component of the electric field, $\mathbf{E} \times \mathbf{n}$, or the magnetic field, $\mathbf{H} \times \mathbf{n}$, are given. Here, we assume that

$$\mathbf{H} \times \mathbf{n}|_{\partial\Omega} = 0 \quad \text{on } \partial\Omega. \tag{1.6}$$