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ON T-SPLINE CLASSIFICATION*

Xin Li[1)] and Liangwei Hong
School of Mathematical Science, USTC, Hefei 230026, China
Email: lixustc@ustc.edu.cn, bodhihlw@mail.ustc.edu.cn

Abstract

The present paper conjectures a topological condition which classifies a T-spline into standard, semi-standard and non-standard. We also provide the basic framework to prove the conjecture on the classification of semi-standard T-splines and give the proof for the semi-standard of bi-degree $(1, d)$ and $(d, 1)$ T-splines.

Mathematics subject classification: 65D07.
Key words: T-splines, Partition of unity, Analysis-suitable, Isogeometric analysis.

1. Introduction

A T-spline surface [1] is defined in terms of a set of control points, weights and blending functions $T_i(s,t)$ as equation (2.1), see more details for the definition of a T-spline in Section 2. Three categories of T-splines are defined in [2]:

1. A semi-standard T-Spline is one for which for any valid choice of knot intervals, there exists a set of weights ω_i such that $\Sigma_{i=1}^{n}\omega_i T_i(s,t) \equiv 1$. Especially, if all the weights are ones, then the T-spline is called standard.

2. A fnon-standard T-Spline is one for which there exists a valid set of knot intervals such that $\Sigma_{i=1}^{n}\omega_i T_i(s,t) \neq 1$ for any choice of weights ω_i.

We address a question posed in [2]: the above definition is "an algebraic statement of necessary and sufficient conditions for a T-spline space to be standard and semi-standard. What is the topological interpretation of those conditions? That is, what T-mesh configurations yield a standard or a semi-standard T-spline?"

Blending function partition of unity is important for both geometric modeling and for isogeometric analysis (for short: IGA) because it assures affine invariance (or, affine covariance in IGA jargon). The set of blending functions for any T-spline, standard or not, forms a partition of unity, if we recognize that there are two definitions for blending functions. The most common definition is that the $T_i(s,t)$ in (2.1) are blending functions. But, (2.1) can be written

$$\mathbf{T}(s,t) = \sum_{i=1}^{n} \frac{\omega_i T_i(s,t)}{\sum_{j=1}^{n}\omega_j T_j(s,t)}\mathbf{C}_i = \sum_{i=1}^{n} R_i(s,t)\mathbf{C}_i$$

and it is clear that the $R_i(s,t)$ always sum to one, regardless of the choice of ω_i and even if the T-spline is non-standard. Affine invariance requires partition of unity of the R_i, not of the T_i.

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1) Corresponding author

For a standard T-spline with all $\omega_i = 1$, $R_i \equiv T_i$ and for a semi-standard T-spline with some weight ω_i such that $R_i \equiv w_i T_i$.

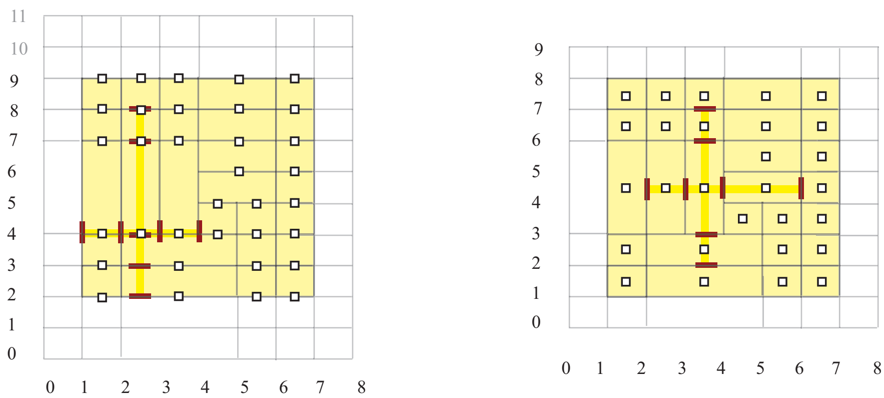
The classification for T-splines are of practical interest because the standard and semi-standard T-splines have several advantages over non-standard T-splines. For example, for some weights, a standard and a semi-standard T-spline can be decomposed into polynomial patches or converted to a polynomial B-spline, making it much simpler than in the rational case to compute partial derivatives and to obtain accurate tessellation error bounds. Also, non-standard T-splines often exhibit wrinkles that are difficult to remove by control point manipulation; this is undesirable for geometric design and also in applications such as surface fitting.

This paper has three main results: Section 3 propose a new conjecture for the classification of arbitrary degree T-splines. Section 4 provides a complete framework for the proof of semi-standard T-splines and Section 5 gives the proof of semi-standard of bi-degree $(1, d)$ (or $(d, 1)$) T-splines. The last section is the conclusion and future work.

2. T-splines

In this section, we prepare some basic notations and preliminary results for arbitrary degree T-splines [3].

2.1. Index T-meshes



c. bi-degree (2,3) d. bi-degree (2,2)

Fig. 2.1. The anchors and the local index vector for one blending function

Similar as the approach in [3], we define T-splines based on the T-meshes in the index domain which are referred as *index T-meshes* in the paper. A T-mesh T for bi-degree (d_1, d_2) T-spline is a connection of all the elements of a rectangular partition of the index domain $[0, c + d_1] \times [0, r + d_2]$, where all rectangle corners (or vertices) have integer coordinates. We define $p = \lfloor \frac{d_1+1}{2} \rfloor$ and $q = \lfloor \frac{d_2+1}{2} \rfloor$, which are the maximal integers equal or less than $\frac{d_1+1}{2}$ and $\frac{d_2+1}{2}$ respectively. Denote the *active region* as rectangle region $[p, c+d_1-p] \times [q, r+d_2-q]$. As we will see below, the active region carries the anchors that will be associated to the blending