Journal of Computational Mathematics
Vol.40, No.3, 2022, 474–485.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2011-m2020-0150

ON T-SPLINE CLASSIFICATION*

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Abstract

The present paper conjectures a topological condition which classifies a T-spline into standard, semi-standard and non-standard. We also provide the basic framework to prove the conjecture on the classification of semi-standard T-splines and give the proof for the semi-standard of bi-degree $(1, d)$ and $(d, 1)$ T-splines.

Mathematics subject classification: 65D07.
Key words: T-splines, Partition of unity, Analysis-suitable, Isogeometric analysis.

1. Introduction

A T-spline surface [1] is defined in terms of a set of control points, weights and blending functions $T_i(s,t)$ as equation (2.1), see more details for the definition of a T-spline in Section 2. Three categories of T-splines are defined in [2]:

1. A semi-standard T-Spline is one for which for any valid choice of knot intervals, there exists a set of weights ω_i such that $\Sigma_{i=1}^{n}\omega_i T_i(s,t) \equiv 1$. Especially, if all the weights are ones, then the T-spline is called standard.

2. A fnon-standard T-Spline is one for which there exists a valid set of knot intervals such that $\Sigma_{i=1}^{n}\omega_i T_i(s,t) \neq 1$ for any choice of weights ω_i.

We address a question posed in [2]: the above definition is "an algebraic statement of necessary and sufficient conditions for a T-spline space to be standard and semi-standard. What is the topological interpretation of those conditions? That is, what T-mesh configurations yield a standard or a semi-standard T-spline?"

Blending function partition of unity is important for both geometric modeling and for isogeometric analysis (for short: IGA) because it assures affine invariance (or, affine covariance in IGA jargon). The set of blending functions for any T-spline, standard or not, forms a partition of unity, if we recognize that there are two definitions for blending functions. The most common definition is that the $T_i(s,t)$ in (2.1) are blending functions. But, (2.1) can be written

$$\mathbf{T}(s,t) = \sum_{i=1}^{n} \frac{\omega_i T_i(s,t)}{\sum_{j=1}^{n}\omega_j T_j(s,t)}\mathbf{C}_i = \sum_{i=1}^{n} R_i(s,t)\mathbf{C}_i$$

and it is clear that the $R_i(s,t)$ always sum to one, regardless of the choice of ω_i and even if the T-spline is non-standard. Affine invariance requires partition of unity of the R_i, not of the T_i.

* Received June 8, 2020 / Revised version received October 9, 2020 / Accepted November 6, 2020 / Published online September 23, 2021 /
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For a standard T-spline with all $\omega_i = 1$, $R_i \equiv T_i$ and for a semi-standard T-spline with some weight ω_i such that $R_i \equiv w_i T_i$.

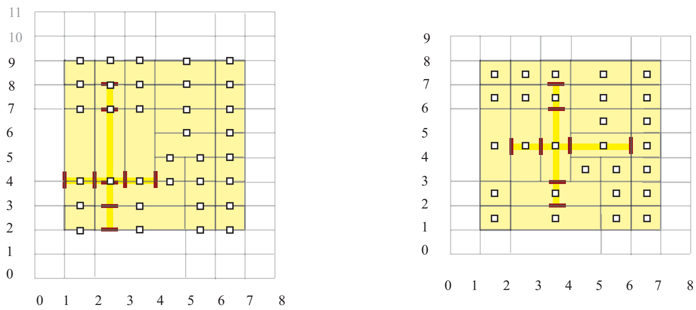
The classification for T-splines are of practical interest because the standard and semi-standard T-splines have several advantages over non-standard T-splines. For example, for some weights, a standard and a semi-standard T-spline can be decomposed into polynomial patches or converted to a polynomial B-spline, making it much simpler than in the rational case to compute partial derivatives and to obtain accurate tessellation error bounds. Also, non-standard T-splines often exhibit wrinkles that are difficult to remove by control point manipulation; this is undesirable for geometric design and also in applications such as surface fitting.

This paper has three main results: Section 3 propose a new conjecture for the classification of arbitrary degree T-splines. Section 4 provides a complete framework for the proof of semi-standard T-splines and Section 5 gives the proof of semi-standard of bi-degree $(1, d)$ (or $(d, 1)$) T-splines. The last section is the conclusion and future work.

2. T-splines

In this section, we prepare some basic notations and preliminary results for arbitrary degree T-splines [3].

2.1. Index T-meshes



c. bi-degree (2,3)   d. bi-degree (2,2)

Fig. 2.1. The anchors and the local index vector for one blending function

Similar as the approach in [3], we define T-splines based on the T-meshes in the index domain which are referred as *index T-meshes* in the paper. A T-mesh T for bi-degree (d_1, d_2) T-spline is a connection of all the elements of a rectangular partition of the index domain $[0, c + d_1] \times [0, r + d_2]$, where all rectangle corners (or vertices) have integer coordinates. We define $p = \lfloor \frac{d_1+1}{2} \rfloor$ and $q = \lfloor \frac{d_2+1}{2} \rfloor$, which are the maximal integers equal or less than $\frac{d_1+1}{2}$ and $\frac{d_2+1}{2}$ respectively. Denote the *active region* as rectangle region $[p, c+d_1-p] \times [q, r+d_2-q]$. As we will see below, the active region carries the anchors that will be associated to the blending functions while the other indices will be needed for the definition of the blending function when the anchor is close to the boundary.

Three type of elements are vertices, edges and faces. The vertex of a rectangle is called the vertex of the T-mesh, which is denoted as (σ_i, τ_i) or $\{\sigma_i\} \times \{\tau_i\}$. An edge is a line segment