Journal of Computational Mathematics
Vol.40, No.4, 2021, 589–606.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2012-m2020-0203

KNOT PLACEMENT FOR B-SPLINE CURVE APPROXIMATION VIA $l_{\infty,1}$-NORM AND DIFFERENTIAL EVOLUTION ALGORITHM *

Jiaqi Luo and Hongmei Kang[1)]
School of Mathematical Sciences, Soochow University, Suzhou 215008, China
Email: khm@suda.edu.cn
Zhouwang Yang[2)]
School of Mathematical Sciences, University of Science and Technology of China, Hefei 230026, China
Email: yangzw@ustc.edu.cn

Abstract

In this paper, we consider the knot placement problem in B-spline curve approximation. A novel two-stage framework is proposed for addressing this problem. In the first step, the $l_{\infty,1}$-norm model is introduced for the sparse selection of candidate knots from an initial knot vector. By this step, the knot number is determined. In the second step, knot positions are formulated into a nonlinear optimization problem and optimized by a global optimization algorithm — the differential evolution algorithm (DE). The candidate knots selected in the first step are served for initial values of the DE algorithm. Since the candidate knots provide a good guess of knot positions, the DE algorithm can quickly converge. One advantage of the proposed algorithm is that the knot number and knot positions are determined automatically. Compared with the current existing algorithms, the proposed algorithm finds approximations with smaller fitting error when the knot number is fixed in advance. Furthermore, the proposed algorithm is robust to noisy data and can handle with few data points. We illustrate with some examples and applications.

Mathematics subject classification: 65D17, 68U07.
Key words: B-spline curve approximation, Knot placement, $l_{\infty,1}$-norm, Differential Evolution algorithm.

1. Introduction

Data fitting with splines is a traditional and classical problem in many fields such as Computer-Aided Design, and geometric modelling. Researchers have found that spline fitting with free knots yields substantial performance improvements. However, knot placement (finding optimal knots) for spline approximation is still a challenging problem due to the highly nonlinear relationship between knots and splines. Many methods have been proposed for addressing this challenging problem computationally, such as nonlinear optimization methods [1–5] and heuristic methods [6–11].

Among the current existing algorithms, the two-stage frameworks [12,13] provide an original idea of computing knots for B-spline functions automatically. In these works, candidate knots are selected in the first step and then adjusted locally in the second step to reduce redundant

* Received July 29, 2020 / Revised version received October 14, 2020 / Accepted December 31, 2020 / Published online October 2, 2021 /
1) Corresponding author
2) Corresponding author

knots. In [13], the unimodality property of initial B-spline approximations is firstly introduced for selecting candidate knots. The initial B-spline approximations are required to approximate given data well enough. B-splines are characterized by the unimodality if jumps from the highest order derivatives of the approximations at some interior knots are local maxima. The unimodality property of initial B-spline approximations gives an adequate estimate of optimal knots for given data. Therefore, the candidate knots selected by the method in [13] can be served as good initial knots for nonlinear optimization methods discussed above. Inspired by this insight, we propose a novel two-stage framework of knot placement for B-spline curve approximation in this paper. In the first step, candidate knots are selected based on the unimodality of initial B-spline curve approximations and served as initial values of the nonlinear optimization problem which is formulated for optimizing knot positions in the second step. We employ the differential evolution (DE) algorithm to solve the nonlinear optimization problem in the second step.

The proposed method is not just a generalization but a great innovation of the two-stage framework introduced in [12, 13]. First, the initial B-spline curve approximations constructed in the first step should demonstrate prominent unimodality property. The l_1-norm model introduced in [12, 13] can not be used here. For this reason, we propose the $l_{\infty,1}$-norm model for constructing initial B-spline curve approximations. Moreover, such a $l_{\infty,1}$-norm model is robust with noisy data and powerful in dealing with few data. Second, the adequate guess of optimal knots provided in the first step and the efficient optimization algorithm employed in the second make it possible to solve the nonlinear optimization problem effectively. Thus our method is easier to get a globally optimized knots than heuristic strategies introduced in [12,13], where candidate knots are adjusted iteratively by heuristic strategies. Here, we choose the DE algorithm for solving the nonlinear optimization problem because it converges to the global optimum faster and is more easy to jump out of local minimums than other algorithms. In conclusion, the DE algorithm together with the candidate knots solved from the $l_{\infty,1}$-norm model facilitates the proposed computationally efficient framework in this paper.

The main contributions of this paper are summarized as follows:

- The knot number and knot positions of B-spline curve approximations are determined automatically.
- Compared with the current existing algorithms, the proposed algorithm finds approximations with smaller fitting error when the knot number is fixed in advance.
- The proposed algorithm is robust to noisy data and can handle with very few data points.

2. Related Work

B-spline curve approximation with fixed knots is a well-studied problem. The approximation problem is formulated into a least squares fitting problem in [14]. In order to avoid the computational cost of solving a large system of linear equations, a progressive and iterative approximation for least square fitting (abbr. LSPIA) is devised for handling point set of large size in [15].

As is well known, approximations with free knots give a better fitting performance. However, challenges come with that. Knots are optimized by a nonlinear optimization problem with the constraint that a knot vector is a nondecreasing sequence. Such a nonlinear optimization problem usually has multiple local minima. In the earlier work [1], the alternative iteration