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TWO-GRID ALGORITHM OF H^1-GALERKIN MIXED FINITE ELEMENT METHODS FOR SEMILINEAR PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS*

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Abstract

In this paper, we present a two-grid discretization scheme for semilinear parabolic integro-differential equations by H^1-Galerkin mixed finite element methods. We use the lowest order Raviart-Thomas mixed finite elements and continuous linear finite element for spatial discretization, and backward Euler scheme for temporal discretization. Firstly, a priori error estimates and some superclose properties are derived. Secondly, a two-grid scheme is presented and its convergence is discussed. In the proposed two-grid scheme, the solution of the nonlinear system on a fine grid is reduced to the solution of the nonlinear system on a much coarser grid and the solution of two symmetric and positive definite linear algebraic equations on the fine grid and the resulting solution still maintains optimal accuracy. Finally, a numerical experiment is implemented to verify theoretical results of the proposed scheme. The theoretical and numerical results show that the two-grid method achieves the same convergence property as the one-grid method with the choice $h = H^2$.

Mathematics subject classification: 49J20, 65N30.

Key words: Semilinear parabolic integro-differential equations, H^1-Galerkin mixed finite element method, A priori error estimates, Two-grid, Superclose.

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1. Introduction

In this work, we shall investigate the following semilinear parabolic integro-differential equations

$$y_t - \mathrm{div}\boldsymbol{p} + \int_0^t \mathrm{div}\boldsymbol{p}\, ds = f(y), \quad (x,t) \in \Omega \times J, \tag{1.1}$$

$$\boldsymbol{p} = \nabla y, \quad (x,t) \in \Omega \times J, \tag{1.2}$$

$$\nabla y \cdot \mathbf{n}|_{\partial\Omega} = 0, \quad (x,t) \in \partial\Omega \times J, \tag{1.3}$$

$$y(x,0) = y_0(x), \quad x \in \Omega, \tag{1.4}$$

where $\Omega \subset \mathbf{R}^2$ is a rectangle with the boundary $\partial\Omega$, $\mathbf{n}$ is the outward unit normal vector to $\partial\Omega$, $J = (0,T]$, $f(y) = f(y,x,t)$ is a given real-valued function on Ω. We assume that

$$|f'(y)| + |f''(y)| \leq M, \quad y \in \mathbf{R}.$$

Integro-differential equations can arise from many physical processes in which it is deficiency (the local characteristic) of the usual diffusion equations. A lot of numerical methods have been developed for solving these problems. Finite element approximation of linear or nonlinear integro-differential equations are extensively studied, see, e.g., [4,5,21,31] for standard finite element method and [13,14,30] for classical mixed finite element method. However, the technique of the classical mixed method leads to some saddle point problems whose numerical solutions have been quite difficult because of losing positive definite properties. To overcome this difficulty, some improved mixed finite element methods were proposed. Pehlivanov et al. [25, 26] developed lease-square mixed finite element method to solve elliptic problems and discussed a priori error estimates. Pani and Gairweather [23, 24] presented H^1-Galerkin mixed finite element method for linear parabolic and parabolic integro-differential equations. A notable advantage of this approach is that the approximating finite element spaces are allowed to be of different polynomial degrees. Yang [36] proposed a splitting positive definite mixed finite element method for miscible displacement of compressible flow in porous media. In the splitting procedure, the mixed element system is symmetric positive definite and the flux equation is separated from pressure equation.

The two-grid method which was first proposed by Xu [33, 34] as a highly efficient discretization method has been widely used to solve nonsymmetric and nonlinear partial differential equations, see, e.g., [19, 22, 29, 35] for standard finite element method, [11, 27] for finite difference method, [1, 6] for finite volume method, [2] for discontinuous Galerkin finite element method and [8, 10, 17, 18] for mixed finite element method. As far as we know, there exist a few works on convergence analysis of two-grid method for parabolic integro-differential equations in the existing literature. Chen et al. [7] presented a two-grid finite element method for a two-dimensional nonlinear parabolic integro-differential equation. For solving parabolic integro-differential equations with nonlinear memory, Wang and Hong [32] designed several two-grid finite element algorithms. Li and Rui [20] introduced a two-grid block-centered finite difference scheme to solve the nonlinear parabolic integro-differential equation arising in modeling non-Fickian flow in porous media. Hou et al. [17] investigated a two-grid discretization scheme for semilinear parabolic integro-differential equations by expanded mixed finite element methods.

Recently, some mixed finite element methods except classical mixed finite element method have been combined with two-grid method to solve nonlinear parabolic problems. Hou et al. [18]