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A DISCRETIZING LEVENBERG-MARQUARDT SCHEME FOR SOLVING NONLIEAR ILL-POSED INTEGRAL EQUATIONS*

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Abstract

To reduce the computational cost, we propose a regularizing modified Levenberg-Marquardt scheme via multiscale Galerkin method for solving nonlinear ill-posed problems. Convergence results for the regularizing modified Levenberg-Marquardt scheme for the solution of nonlinear ill-posed problems have been proved. Based on these results, we propose a modified heuristic parameter choice rule to terminate the regularizing modified Levenberg-Marquardt scheme. By imposing certain conditions on the noise, we derive optimal convergence rates on the approximate solution under special source conditions. Numerical results are presented to illustrate the performance of the regularizing modified Levenberg-Marquardt scheme under the modified heuristic parameter choice.

Mathematics subject classification: 65J15, 65J20, 65J22.
Key words: The regularizing Levenberg-Marquardt scheme, Multiscale Galerkin methods, Nonlinear ill-posed problems, Heuristic parameter choice rule, Optimal convergence rate.

1. Introduction

Many researchers have achieved a lot of important results in nonlinear ill-posed problems, see, e.g., [16,18,19,21,24,29,36,46]. However, the research results on numerical implementation in nonlinear ill-posed problems are very limited [23,30]. In this paper we will fill in this gap by developing a regularizing modified Levenberg-Marquardt scheme via multiscale Galerkin method. Generally, in order to perform numerical calculations, we have to transform the problem of infinite dimensionality into a finite dimensional problem [37,38,44]. Therefore, the study of infinite to finite dimension is of great significance. Among conventional numerical methods for solving ill-posed problems, the collocation method [31,42] and the Galerkin method [1,22,40] are well known. The collocation method has received the most favorable attention in ill-posed problems due to its lower computational cost, in comparison with the Galerkin method. In this paper, we will show that the Galerkin method also has its own advantages (see (4.4)). When the projection space is determined, the influence of noise can be reduced by the Galerkin method, e.g., denoising by the wavelet basis.

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Although multiscale Galerkin method [5, 6, 8, 9] for solving linear ill-posed problems have been widely used [10, 11, 34, 35, 43], less attention has been paid to the development of multiscale Galerkin method for solving nonlinear ill-posed problems [4]. One of the bottlenecks of multiscale Galerkin method for solving nonlinear ill-posed problems is the huge computational cost. In nonlinear regularization methods [13], compared with the linear ill-posed problem, for solving the nonlinear problem with multiscale Galerkin method, we need to keep updating the coefficient matrix. Note that the amount of computation required to update the coefficient matrix is huge. Hence, we propose a regularizing modified Levenberg-Marquardt scheme to solving nonlinear ill-posed problems. Such an idea has already been used in [46]. Obviously, we can directly use the two methods mentioned in [46]. Unfortunately, the number of iterations for both methods is not satisfactory. Fortunately, we found that the regularizing Levenberg-Marquardt scheme is very fast and effective in [18, 19, 24]. That is why we proposed the regularizing modified Levenberg-Marquardt scheme to solve nonlinear ill-posed problems.

The parameter choice rules are mainly divided into three categories: a prior parameter choice rule [14, 20, 39], a posterior parameter choice rule [21, 26, 36, 41, 47, 48], and heuristic parameter choice rule [17, 25, 28, 29, 50]. The prior parameter choice rule and the posterior parameter rule require accurate knowledge of the noise level to obtain satisfactory approximate solutions. The heuristic parameter selection criterion is not guaranteed to always converge [2], but the heuristic parameter selection criterion does not require such noise level information. In practical applications, such noise level is not always available or reliable. Therefore, the more popular ones is the heuristic parameter choice rule.

This paper is organized as follows. In Section 2, we shall propose a modified Levenberg-Marquardt scheme via multiscale Galerkin methods to solve nonlinear ill-posed problems. The modified Levenberg-Marquardt scheme greatly reduces the amount of calculations originally performed. In Section 3 we prove the convergence of the modified Levenberg-Marquardt scheme via multiscale Galerkin methods for the noise free case. In Section 4 we prove convergence of the modified Levenberg-Marquardt scheme via multiscale Galerkin methods for the noise case. In Section 5, we propose a modified heuristic parameter choice rule under these convergence results. Based on certain conditions on the noise, we prove the optimal convergence rate of the approximate solution. Finally, in Section 6 we report some numerical results to verify the theoretical analysis for the modified heuristic parameter choice rule.

2. A Regularizing Modified Levenberg-Marquardt Scheme

In this section, we propose a regularizing modified Levenberg-Marquardt scheme via multiscale Galerkin methods for solving the nonlinear ill-posed problems. By comparing it with the multilevel augmentation methods, we can know that such scheme greatly reduces the amount of numerical calculation.

We shall deal with the nonlinear integral equation

$$F(x) = y \tag{2.1}$$

arising from ill-posed problems [13, 32], where operator $F : \mathcal{D}(F) \subset \mathbb{X} \to \mathbb{Y}$ is Fréchet differentiable and given by

$$F(x) := \int_{\Omega} k(s, t, x(t))dt, \quad s \in \Omega \subset \mathbb{R},$$

where $k(s, t, x(t))$ is nonlinear kernel, $x(t)$ is unknown function and $\mathbb{R}$ is the set of real numbers. In many applications it follows from physical considerations that y^{δ} we obtain is a reasonably