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PRIMAL-DUAL PATH-FOLLOWING METHODS AND THE TRUST-REGION UPDATING STRATEGY FOR LINEAR PROGRAMMING WITH NOISY DATA*

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Abstract

In this article, we consider the primal-dual path-following method and the trust-region updating strategy for the standard linear programming problem. For the rank-deficient problem with the small noisy data, we also give the preprocessing method based on the QR decomposition with column pivoting. Then, we prove the global convergence of the new method when the initial point is strictly primal-dual feasible. Finally, for some rank-deficient problems with or without the small noisy data from the NETLIB collection, we compare it with other two popular interior-point methods, i.e. the subroutine pathfollow.m and the built-in subroutine linprog.m of the MATLAB environment. Numerical results show that the new method is more robust than the other two methods for the rank-deficient problem with the small noise data.

Mathematics subject classification: 65L20, 65K05, 65L05.
Key words: Continuation Newton method, Trust-region method, Linear programming, Rank deficiency, Path-following method, Noisy data.

1. Introduction

In this article, we are mainly concerned with the linear programming problem with the small noisy data as follows:

$$\min_{x\in\Re^n} c^T x, \quad \text{subject to} \quad Ax = b, \quad x \geq 0, \tag{1.1}$$

where c and x are vectors in $\Re^n$, b is a vector in $\Re^m$, and A is an $m \times n$ matrix. For the problem (1.1), there are many efficient methods to solve it such as the simplex methods [38,48], the interior-point methods [18,21,36,43,45,49] and the continuous methods [1,11,26,34]. Those methods are all assumed that the constraints of problem (1.1) are consistent, i.e. $\text{rank}(A, b) = \text{rank}(A)$. For the consistent system of redundant constraints, references [3, 4, 33] provided a few preprocessing strategies which are widely used in both academic and commercial linear programming solvers.

However, for a real-world problem, since it may include the redundant constraints and the measurement errors, the rank of matrix A may be deficient and the right-hand-side vector b has small noise. Consequently, they may lead to the inconsistent system of constraints [6, 12, 29]. On the other hand, the constraints of the original real-world problem are intrinsically consistent. Therefore, we consider the least-squares approximation of the inconsistent constraints in the linear programming problem based on the QR decomposition with column pivoting. Then,

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according to the first-order KKT conditions of the linear programming problem, we convert the processed problems into the equivalent problem of nonlinear equations with nonnegative constraints. Based on the system of nonlinear equations with nonnegative constraints, we consider a special continuous Newton flow with nonnegative constraints, which has the nonnegative steady-state solution for any nonnegative initial point. Finally, we consider a primal-dual path-following method and the adaptive trust-region updating strategy to follow the trajectory of the continuous Newton flow. Thus, we obtain an optimal solution of the original linear programming problem.

The rest of this article is organized as follows. In the next section, we consider the primal-dual path-following method and the adaptive trust-region updating strategy for the linear programming problem. In Section 3, we analyze the global convergence of the new method when the initial point is strictly primal-dual feasible. In Section 4, for the rank-deficient problems with or without the small noise, we compare the new method with two other popular interior-point methods, i.e. the traditional path-following method (pathfollow.m in [18, p.210]) and the predictor-corrector algorithm (the built-in subroutine linprog.m of the MATLAB environment [31, 32, 49]). Numerical results show that the new method is more robust than the other two methods for the rank-deficient problem with the small noisy data. Finally, some discussions are given in Section 5. $\|\cdot\|$ denotes the Euclidean vector norm or its induced matrix norm through the paper.

2. Primal-Dual Path-Following Methods and the Trust-Region Updating Strategy

2.1. The continuous Newton flow

For the linear programming problem (1.1), it is well known that x^* is its optimal solution if and only if it satisfies the following Karush-Kuhn-Tucker conditions [36, pp.396-397]:

$$Ax - b = 0, \quad A^T y + s - c = 0, \quad XSe = 0, \quad (x, s) \geq 0, \tag{2.1}$$

where

$$X = \mathrm{diag}(x), \quad S = \mathrm{diag}(s), \quad e = (1, \cdots, 1)^T. \tag{2.2}$$

For convenience, we rewrite the optimality condition (2.1) as the following nonlinear system of equations with nonnegative constraints:

$$F(z) = \begin{bmatrix} Ax - b \\ A^T y + s - c \\ XSe \end{bmatrix} = 0, \quad (x, s) \geq 0, \quad \text{and} \quad z = (x, y, s). \tag{2.3}$$

It is not difficult to know that the Jacobian matrix $J(z)$ of $F(z)$ has the following form:

$$J(z) = \begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ S & 0 & X \end{bmatrix}. \tag{2.4}$$

From the third block $XSe = 0$ of Eq. (2.3), we know that $x_i = 0$ or $s_i = 0$ $(i = 1 : n)$. Thus, the Jacobian matrix $J(z)$ of Eq. (2.4) may be singular, which leads to numerical difficulties near