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A CHARACTERISTIC MIXED FINITE ELEMENT TWO-GRID METHOD FOR COMPRESSIBLE MISCIBLE DISPLACEMENT PROBLEM*

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Abstract

A nonlinear parabolic system is derived to describe compressible miscible displacement in a porous medium. The concentration equation is treated by a mixed finite element method with characteristics (CMFEM) and the pressure equation is treated by a parabolic mixed finite element method (PMFEM). Two-grid algorithm is considered to linearize nonlinear coupled system of two parabolic partial differential equations. Moreover, the L^q error estimates are conducted for the pressure, Darcy velocity and concentration variables in the two-grid solutions. Both theoretical analysis and numerical experiments are presented to show that the two-grid algorithm is very effective.

Mathematics subject classification: 65M06, 65M12, 65M15, 65M55.
Key words: Two-grid method, Miscible displacement problem, Mixed finite element, Characteristic finite element method.

1. Introduction

Compressible miscible displacement problems in a porous medium are fundamental processes arising in many diversified fields such as petroleum engineering, groundwater hydrology, and environmental engineering. The mathematical models used to describe miscible displacement problem are coupled systems of nonlinear parabolic partial differential equations. Chen and Ewing [1] have discussed the mathematic theory of these models.

The miscible displacement of one compressible fluid by another in a reservoir $\Omega \subset \mathbb{R}^2$ of unit thickness is considered. The nonlinear coupled system of two parabolic partial differential equations is given by

$$\phi\frac{\partial c}{\partial t} + b(c)\frac{\partial p}{\partial t} + \boldsymbol{u}\cdot\nabla c - \nabla\cdot(\boldsymbol{D}\nabla c) = f(c), \tag{1.1}$$

$$d(c)\frac{\partial p}{\partial t} + \nabla\cdot\boldsymbol{u} = q, \tag{1.2}$$

$$\boldsymbol{u} = -a(c)\nabla p, \tag{1.3}$$

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where $x \in \Omega$, $t \in J = [0, T]$, the dependent variables p and $\boldsymbol{u}(x,t)$ are the pressure in the fluid mixture and the Darcy velocity of the mixture, respectively. $c(x,t)$ is the solvent concentration of interested species measured in amount of species per unit volume of the fluid mixture. $\phi(x)$ is the effective porosity. q is a sum of sources (injection) and sinks (extraction) and is assumed to be bounded. $f(c)$ is nonlinear function. $c_0(x)$ is the initial concentration. $a(c) = \frac{k(x)}{\mu(c)}$, where $k(x)$ is the permeability of the porous rock and $\mu(c)$ is the viscosity of the fluid mixture. The dispersion-diffusion tensor $\boldsymbol{D}(\boldsymbol{u})$ has contributions from molecular diffusion and mechanical dispersion and it is the 2×2 matrix,

$$\boldsymbol{D} = \phi[d_m \boldsymbol{I} + |\boldsymbol{u}|(d_l \boldsymbol{E}(u) + d_t \boldsymbol{E}^{\perp}(\boldsymbol{u}))],$$

where $(\boldsymbol{E}(\boldsymbol{u}))_{i,j} = \frac{\boldsymbol{u}_i \boldsymbol{u}_j}{|\boldsymbol{u}|^2}$ and $\boldsymbol{E}^{\perp} = \boldsymbol{I} - \boldsymbol{E}$, d_m is the molecular diffusion, and d_l, d_t are, respectively, the longitudinal and transverse dispersion coefficients. For simplicity, we assume that $\boldsymbol{D} = \phi d_m \boldsymbol{I}$ implies only the molecular diffusion in this paper.

The initial conditions and the boundary conditions are

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \qquad x \in \partial\Omega, \quad t \in J, \tag{1.4}$$

$$\boldsymbol{D}\nabla c \cdot \boldsymbol{n} = 0, \qquad x \in \partial\Omega, \quad t \in J, \tag{1.5}$$

$$c(x,0) = c_0(x), \qquad x \in \Omega, \tag{1.6}$$

$$p(x,0) = p_0(x), \qquad x \in \Omega. \tag{1.7}$$

The coefficients $b(c)$ and $d(c)$ are defined as:

$$b(c) = \phi(x) c_1 \left\{ z_1 - \sum_{j=1}^{2} z_j c_j \right\}, \quad d(c) = \phi(x) \sum_{j=1}^{2} z_j c_j,$$

where $c = c_1 = 1 - c_2$, c_i denotes the (volumetric) concentration of the ith component of the fluid mixture, $i = 1, 2$. z_j is the constant compressibility factor for the jth component, $j = 1, 2$.

We assume that the functions $b(c)$ and $d(c)$ are twice continuously differentiable with bounded derivatives through the second order.

We also require the following assumptions on the coefficients in (1.1)-(1.3). Let a_*, a^*, ϕ_*, ϕ^*, a_0 and C be positive constants such that

$$a_* \le a(c) \le a^*, \qquad \phi_* \le \phi(x) \le \phi^*,$$

$$\sum_{i,j=1}^{2} \boldsymbol{D}_{i,j} \xi_i \xi_j \ge a_0 |\xi|^2, \qquad \forall \xi \in \mathbb{R}^2,$$

$$\left\| \frac{\partial c}{\partial t} \right\| + \left\| \frac{\partial^2 c}{\partial t^2} \right\| + \|q\|_{L^\infty} \le M.$$

Furthermore, we suppose that the functions $\alpha(c) = a^{-1}(c)$ and $\boldsymbol{\kappa}(\boldsymbol{u}) = \boldsymbol{D}^{-1}(\boldsymbol{u})$ are twice continuously differentiable with bounded derivatives through the second order.

In the past thirty years, numerical approximations have received a great deal of attention for the miscible displacement of one incompressible and compressible fluid by another in a reservoir [8–10, 13, 14, 21–26]. Douglas and his coauthors [9] employed Galerkin methods and mixed finite element methods to approximate compressible miscible displacement for continuous time approximation. Cheng [4] studied the optimal error estimates of time-discretization approximation to compressible miscible displacement problem. It is well known that the physical