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THE A POSTERIORI ERROR ESTIMATOR OF SDG METHOD FOR VARIABLE COEFFICIENTS TIME-HARMONIC MAXWELL'S EQUATIONS*

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Abstract

In this paper, we study the a posteriori error estimator of SDG method for variable coefficients time-harmonic Maxwell's equations. We propose two a posteriori error estimators, one is the recovery-type estimator, and the other is the residual-type estimator. We first propose the curl-recovery method for the staggered discontinuous Galerkin method (SDGM), and based on the super-convergence result of the postprocessed solution, an asymptotically exact error estimator is constructed. The residual-type a posteriori error estimator is also proposed, and it's reliability and effectiveness are proved for variable coefficients time-harmonic Maxwell's equations. The efficiency and robustness of the proposed estimators is demonstrated by the numerical experiments.

Mathematics subject classification: 65N30, 65F10.
Key words: Maxwell's equations, A posteriori error estimation, Staggered discontinuous Galerkin.

1. Introduction

In recent decades, the discontinuous finite element method, with its high flexibility in adapting complex geometries and independence in selecting the polynomial on each element, has significant advantages for solving complex partial differential systems. In the early 1970s, different types of discontinuous finite element methods were developed for many different partial differential problems. The discontinuous finite element method in the form of numerical flux initially originated from solving the first-order hyperbolic problem [1], and the discontinuous finite element method in the form of penalty was derived from solving the elliptic problem [2]. Subsequently, a certain internal connection was established between the numerical flux type discontinuous finite element method and penalty type discontinuous finite element method [3]. In the process of development, various types of DG methods have emerged and are widely

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used in the fields of physics and engineering. For example, aerodynamics [9], semiconductor devices [10, 11], electromagnetics [12, 15–17], etc. In recent years, DG method also plays an important role in the calculation of electromagnetic problems. In a series of articles [15–17], an interior penalty discontinuous Galerkin method for the indefinite time-harmonic Maxwell's equations was proposed and proved the optimal error estimate of the energy norm and L^2 norm, and the incorporation of divergence-free constraint in DG framework was handled by a suitable Lagrange multiplier. For the hybridizable discontinuous finite element method of the time-harmonic Maxwell's equations at low frequency case, Nguyen et al. [12] also introduced the Lagrange multiplier to ensure the stability of the format, and used the post-processing technique based on the k-degree polynomial approximation. A new H^{curl} conforming approximate solution is obtained, and its H^{curl} norm can reach k+1 order accuracy. In 2004, Lu et.al proposed the time-domain DG method for the first-order Maxwell's equation in dispersive media [18]. In 2009, the internal penalty DG method was proposed for Maxwell's equations in dispersive media, and the a priori error estimate [19] and the a posteriori error estimate [20] of this method were obtained. The energy norm error estimate has reached the optimal convergence order in [19]. Subsequently, Li proposed the L^2 norm optimal error estimate for the semi-discrete DG scheme and the fully explicit DG format [21].

E. T. Chung et al. proposed a new type of discontinuous finite element method [22–24] when solving the wave propagation problem, which is the so-called staggered discontinuous galerkin method. It not only satisfies the explicitly format, energy conservation, but also achieves the optimal convergence order accuracy. Afterwards, the staggered discontinuous galerkin method was used to solve problems such as convection-diffusion [25], Stokes system [29], and curl-curl operator [26], and significant results were obtained. When the discontinuous finite element itself solves the time-development problem, the time explicit integration format can make it generate a diagonal mass matrix, which can achieve the effect of elementwise decoupling. It is suitable for parallel calculation, which reduces the computational time and improves the efficiency. At the same time, the discrete forms of curl operators produced by the staggered discontinuous galerkin method are accompanied by each other, so they inherit some conservation properties from the original differential equation. In addition, when solving partial differential equations that have divergence-free solutions, the staggered discontinuous galerkin method is different from other discontinuous finite element methods, which need to restrict divergence-free condition to avoid spurious solutions, because numerical solution of the staggered discontinuous galerkin method satisfy automatically the discrete divergence-free condition [26].

When solving partial differential equations in practical problems, most of the solutions contain strong singularities. For the time-harmonic Maxwell's equation, various factors such as non-trivial geometry, discontinuous parameters and non-smooth source terms will lead to singular solutions. [30, 31] Uniform refinement mesh will lead to a sharp increase in computational cost. Therefore, adaptive algorithms are generally used to capture singularities and locally refine the grid to ensure good accuracy of the solution. In recent years, many scholars have conducted in-depth research on the residual type a posteriori error estimation of discontinuous finite element methods [7, 32]. In particular, E. T. Chung et al. [14] analyzed the residual type a posteriori error for the staggered discontinuous galerkin method of the constant coefficient time-harmonic Maxwell's equations. There are other types of a posteriori error estimators for time-harmonic Maxwell's equations. For example, recovery type a posteriori error estimator can be constructed by using the superconvergence property of the derivative of the numerical solution at some special points on the element. Recently, we have proposed a recovery type a