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A FINITE VOLUME METHOD PRESERVING MAXIMUM PRINCIPLE FOR THE CONJUGATE HEAT TRANSFER PROBLEMS WITH GENERAL INTERFACE CONDITIONS*

Huifang Zhou
School of Mathematics, Jilin University, Changchun 130012, China
The Graduate School of China Academy of Engineering Physics, Beijing 100088, China
Email: 13614405274@163.com
Zhiqiang Sheng
Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China
HEDPS, Center for Applied Physics and Technology, and College of Engineering, Peking University, Beijing 100871, China
Email: sheng_zhiqiang@iapcm.ac.cn
Guangwei Yuan[1)]
Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China
Email: yuan_guangwei@iapcm.ac.cn

Abstract

In this paper, we present a unified finite volume method preserving discrete maximum principle (DMP) for the conjugate heat transfer problems with general interface conditions. We prove the existence of the numerical solution and the DMP-preserving property. Numerical experiments show that the nonlinear iteration numbers of the scheme in [24] increase rapidly when the interfacial coefficients decrease to zero. In contrast, the nonlinear iteration numbers of the unified scheme do not increase when the interfacial coefficients decrease to zero, which reveals that the unified scheme is more robust than the scheme in [24]. The accuracy and DMP-preserving property of the scheme are also verified in the numerical experiments.

Mathematics subject classification: 65M08, 35K59.
Key words: Conjugate heat transfer problems, General interface conditions, Finite volume scheme, Discrete maximum principle.

1. Introduction

The conjugate heat transfer refers to the phenomenon of thermal interaction between different materials with different temperature. The problems are often illustrated by elliptic or parabolic interface problems in partitioned domain, in which the interface represents where the contact occurs. The conjugate heat transfer arises widely in physics and engineering, such as the modeling of heat transfer through multilayered walls in buildings, electronics packaging, heat exchangers, space craft structures and nuclear reactor [15], etc.

Thermal properties of different materials, such as thermal conductivity, heat capacity, and density, are usually different, which causes discontinuity of diffusion coefficients across the

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1) Corresponding author

interface. Across the interface, these problems also impose the temperature and the conductive heat flux satisfying certain interface conditions. According to the interface conditions, the elliptic interface problems can be divided into the following two types.

The most common problems are the perfect interface problems, in which both the temperature and the conductive heat flux are required to be continuous. The corresponding conditions are called perfect interface conditions. This kind of problems is usually solved as an overall problems on the whole domain. There have been many researches concerning about this problem, such as finite difference methods [9], finite element methods [12, 25], finite volume methods [14, 17, 18, 21], and discontinuous Galerkin (DG) methods [5, 13, 22], and so on.

The other type is the imperfect interface problems. When the two materials are relatively sliding, there exists interfacial heat resistance, which is called Kapitza resistance, impeding the heat transfer across the interface. In this case, the conductive heat flux is still conservative, while the temperature is discontinuous because of the roughness of the interface. The model of perfect interface problems is not suitable for such kind of problems any more. For this type of problems, the jump of temperature is usually in proportion to the normal heat flux, which is known as imperfect interface conditions. Moreover, the proportion is denoted by $\mu(\mathbf{x})$ in the rest of paper, which is always non-negative. The coefficient is also called interfacial thermal resistance coefficient or Kapitza coefficient.

Many numerical methods only concern about problems with positive interfacial coefficient $\mu > 0$, such as finite element methods [8,10], finite difference methods [3], finite volume methods [2, 4, 24], DG Methods [1, 7]. For the finite element methods, a DMP-preserving method [8] and a nonstandard variation form [10] are proposed. For the finite volume methods, high-order scheme [4], positivity-preserving scheme [24] and DMP-preserving scheme [2] are studied, respectively. An interior-penalty DG method [1] and local DG Method [7] are presented.

However, the actual materials on interface may be variant, therefore the corresponding interfacial coefficients may be zero somewhere and non-zero elsewhere. On one hand, the coefficient matrices of schemes designed for imperfect interface problems [1–4,7,8,10,24] are close to singular when the interfacial coefficients tend to zero, which leads to efficiency decrease of the scheme. On the other hand, the above schemes are ill-posed for the perfect interface problems when interfacial coefficients degenerate to zero somewhere. Based on the above considerations, it is crucial to propose a unified scheme to deal with the interface problems with general contact conditions, which means that the scheme is effective both for the perfect interface problems and the imperfect interface problems. Unfortunately, limited work has been done in unified discretization for such problems. To the best of our knowledge, the unified scheme for general interfacial coefficient is only proposed in [16, 19, 20]. A non-traditional finite element method is proposed on non-body-fitting grids in [19]. A special iterative method which is robust with respect to the interfacial coefficient is designed in [20]. In [16], both continuous variational formulation and the finite element method are considered, and the numerical results are compared with the real optical micrograph. There also exist some researches concerning other types of interface problem, where the jumps of solution and flux on the interface are given functions. In [11], a finite difference method is proposed on Cartesian grids. In [6], a symmetric discontinuous Galerkin method is studied on fitted meshes, and the high-order convergence is proved.

The maximum principle is an essential property for the conjugate heat transfer problems and it reveals physical restriction of unknowns, such as temperature. If a numerical scheme does not preserve the DMP, it may produce non-physical oscillation and even cause calculation