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ANISOTROPIC EQ_1^{ROT} FINITE ELEMENT APPROXIMATION FOR A MULTI-TERM TIME-FRACTIONAL MIXED SUB-DIFFUSION AND DIFFUSION-WAVE EQUATION*

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Abstract

By employing EQ_1^{rot} nonconforming finite element, the numerical approximation is presented for multi-term time-fractional mixed sub-diffusion and diffusion-wave equation on anisotropic meshes. Comparing with the multi-term time-fractional sub-diffusion e-quation or diffusion-wave equation, the mixed case contains a special time-space coupled derivative, which leads to many difficulties in numerical analysis. Firstly, a fully discrete scheme is established by using nonconforming finite element method (FEM) in spatial direction and L1 approximation coupled with Crank-Nicolson (L1-CN) scheme in temporal direction. Furthermore, the fully discrete scheme is proved to be unconditional stable. Besides, convergence and superclose results are derived by using the properties of EQ_1^{rot} nonconforming finite element. What's more, the global superconvergence is obtained via the interpolation postprocessing technique. Finally, several numerical results are provided to demonstrate the theoretical analysis on anisotropic meshes.

Mathematics subject classification: 65N15, 65N30, 65M60.
Key words: Multi-term time-fractional mixed sub-diffusion and diffusion-wave equation, Nonconforming FEM, L1-CN scheme, Anisotropic meshes, Convergence and superconvergence.

1. Introduction

In the last few years, fractional calculus have gained much attraction due to the ability to model many anomalous phenomena and complex systems in various scientific fields. Many researchers focused on fractional models and their applications. For example, [1] built the fractional water wave model including fractional derivative in temporal direction by virtue of a

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new time two-mesh FEM. Based on an effective predictor-corrector method, [2] investigated the multi-term time-fractional Bloch equations with respect to the anomalous relaxation processes. In [3], Wang et al. solved a series of fractional diffusion equations and found that these equations can offer a new reference for economic development. After analysing the signal propagation in the electromagnetic media described by the fractional-order models (FOMs), Stenfański et al. [4] found FOMs are better than the integer-order ones. Three transport models based on fractional equations were developed and their applicabilities through experiments were demonstrated in [5]. Analytical solutions of fractional partial differential equations (FPDEs) were studied by some scholars via the method of variables separation and other methods [6–10]. The analytical solutions often contains some special functions, such as multi-variable Mittag-Leffler function. Hence numerical methods for FPDEs have gained considerable popularity in the recent years.

Currently, some scholars investigated numerical methods for the multi-term time-fractional sub-diffusion equation and time-fractional diffusion-wave equation, which are two important kinds of multi-term time-fractional partial differential equation. For example, spectral method [11, 12], FEM [13, 14], orthogonal spline collocation method [15], compact exponential difference method [16], local discontinuous Galerkin method [17], Crank-Nicolson finite difference method (FDM) combined with exponential B-spline method [18], FDM jointed with matrix transfer method [19] were discussed for multi-term time-fractional sub-diffusion equation. In detail, in [12], by combining L1 method on graded meshes and Legendre spectral method, Zheng et al. solved the two-dimensional multi-term time-fractional diffusion equation with non-smooth solutions. Based on a standard Galerkin FEM, semi-discrete and fully discrete schemes were established for the multi-term time-fractional diffusion equation in [14]. In [16], a compact exponential FEM was considered to solve multi-term time-fractional convection-reaction-diffusion equation and the optimal error estimate under the non-smooth data was obtained. Furthermore, by applying FDM in time and exponential B-spline method in space, an implicit numerical scheme for the multi-term time-fractional diffusion equation was introduced in [18]. In [19], a numerical approach based on the matrix transfer method for solving the multi-term time-fractional diffusion equation was proposed. For the multi-term time-fractional diffusion-wave equation, semi-analytical collocation Trefftz method [20], difference method [21], Gauss-Lobatto-Legendre-Birkhoff pseudospectral method [22], wavelet method [23], fast linearized FDM [24], FEM [25] have been developed. Specially, a semi-analytical collocation Trefftz scheme was considered for multi-term time fractional diffusion-wave equation in [20], which was based on the Laplace transformation and the composite multiple reciprocity method. Two temporal second-order schemes for the time multi-term fractional wave equation were proposed in [21], and the stability and convergence of the two schemes were investigated rigorously. Liu et al. [22] studied the multi-term time fractional diffusion-wave equation with Neumann boundary conditions by the method of a second-order pseudospectral. Lyu et al. [24] investigated the nonlinear multi-term time-fractional wave equation by using a fast linearized FDM and analyzed the truncation error.

Multi-term time-fractional mixed sub-diffusion and diffusion-wave equation is a better choice to describe some underlying processes, which is more precisely than multi-term time-fractional sub-diffusion equation or time-fractional diffusion-wave equation. As far as we know, there are limited literature about the numerical solutions of multi-term time-fractional mixed sub-diffusion and diffusion-wave equation [26–30]. For example, by using linear triangle FEM in spatial direction and L1 time-stepping method jointed with Crank-Nicloson scheme in temporal direction, a fully discrete scheme was established for the 2D multi-term time-fractional mixed