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THE NONCONFORMING CROUZEIX-RAVIART ELEMENT APPROXIMATION AND TWO-GRID DISCRETIZATIONS FOR THE ELASTIC EIGENVALUE PROBLEM*

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Abstract

In this paper, we extend the work of Brenner and Sung [Math. Comp. 59, 321–338 (1992)] and present a regularity estimate for the elastic equations in concave domains. Based on the regularity estimate we prove that the constants in the error estimates of the nonconforming Crouzeix-Raviart element approximations for the elastic equations/eigenvalue problem are independent of Lamé constant, which means the nonconforming Crouzeix-Raviart element approximations are locking-free. We also establish two kinds of two-grid discretization schemes for the elastic eigenvalue problem, and analyze that when the mesh sizes of coarse grid and fine grid satisfy some relationship, the resulting solutions can achieve the optimal accuracy. Numerical examples are provided to show the efficiency of two-grid schemes for the elastic eigenvalue problem.

Mathematics subject classification: 65N25, 65N30.
Key words: Elastic eigenvalue problem, Nonconforming Crouzeix-Raviart element, Two-grid discretizations, Error estimates, Locking-free.

1. Introduction

Due to the wide application background, the approximate computation for elastic equations/eigenvalue problems has attracted the attention of academic circles, for instance, [5, 10, 11, 21, 24, 29, 30, 32, 33, 36, 37, 39–41, 45–47, 54], etc. It is known that for numerical solutions of the equations of linear isotropic planar elasticity, standard conforming finite elements suffer a deterioration in performance as the Lamé constant $\lambda \to \infty$, that is locking phenomenon (see [4,5]). To overcome the locking phenomenon, several numerical approaches have been developed. For example, the p-version method [44], the PEERS method [1], the mixed method [43], the Galerkin least squares method [23], the nonconforming triangular elements [11, 21] and quadrilateral elements [32, 37, 47, 54], and so on.

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For the computation of eigenvalue problems in elasticity, there have been quite a few studies. For instance, [40] adopts a preconditioning technique associated with dimensional reduction algorithm for the thin elastic structures. [39] presents a method for three-dimensional linear elasticity or shell problems to derive computable estimates of the approximation error in eigenvalues. [45] develops an a posteriori error estimator for linearized elasticity eigenvalue problems. [29] analyzes the finite element approximation of the spectral problem for the linear elasticity equation with mixed boundary conditions in a curved concave domain. [36] conducts an analysis for the eigenvalue problem of linear elasticity by means of a mixed variational formulation. [41] presents a theory for the approximation of eigenvalue problems in mixed form by nonconforming methods and apply it to the classical Hellinger-Reissner mixed formulation for a linear elastic structure, etc. Recently, [33] uses the immersed finite element method based on Crouzeix-Raviart (C-R) P1-nonconforming element to approximate eigenvalue problems for elasticity equations with interfaces. [24] explores a shifted-inverse adaptive multigrid method for the elastic eigenvalue problem.

In the above literatures, [10, 11, 21, 33] study the nonconforming C-R element method for the elastic equations/eigenvalue problems in convex domains, and as far as we know, there is no report on the nonconforming C-R approximation for the elastic eigenvalue problems in concave domain. In this paper, we extend the work in [10, 11] and present a regularity estimate for the elastic equations in concave domain (see (2.8)). Since in the standard error analysis for the consistency term, it is required that the "minimum" regularity $\mathbf{u} \in \mathbf{H}^{1+s}(\Omega)$ for $s \geq 1/2$ which is not necessarily satisfied in concave domain, [28, 35] adopt a new method to conduct the error estimate for the C-R element approximation. To be more specific, they made use of the conforming interpolation of the nonconforming C-R element approximation. However, at present we cannot use their method to warrant the error estimates are locking-free for the elastic eigenvalue problem. So, we adopt the argument in [6, 13] to prove a trace inequality in which the constant is analyzed elaborately (see Lemma 3.3) with the condition slightly different from that in the existing literatures and then derive the estimates of consistency term. Based on the regularity estimate we prove that the constants in the error estimates of the nonconforming C-R element approximations for the elastic equations/eigenvalue problem are independent of the Lamé constant, which means the C-R element approximations are locking-free.

Since introduced by Xu and Zhou [49, 50], due to the good performance in reducing computational costs and improving accuracy, the two-grid discretization method has been developed and successfully applied to other problems, for instance, Poisson equation/integral equation eigenvalue problems [51, 52], semilinear eigenvalue problem [16], Stokes equations [12, 34, 38], Schrödinger equation [15, 25], quantum eigenvalue problem [20], Steklov eigenvalue problem [7, 48] and so on. In this paper, we establish two kinds of two-grid discretization schemes of nonconforming C-R element. We prove that the constants in error estimates are independent of the Lamé constants, i.e., the two-grid discretization schemes of nonconforming C-R element are also locking-free, and when the mesh sizes of coarse grid and fine grid satisfy some relationship, the resulting solutions can achieve the optimal accuracy. We present some numerical examples to show the two-grid discretization schemes are efficient for solving elastic eigenvalue problem.

The rest of the paper is organized as follows. Some preliminaries are given in Section 2. The nonconforming C-R element approximation for the elastic eigenvalue problem is established in Section 3. Two-grid discretization schemes and the corresponding error analysis are presented in Section 4. Finally, numerical experiments are shown in Section 5.

We refer to [3, 8, 10, 17] as regards the basic theory of finite element methods in this paper.