AN APPROACH TO FINDING A GLOBAL MINIMIZATION WITH EQUALITY AND INEQUALITY CONSTRAINTS*

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Abstract

We give an approch for finding a global minimization with equality and inequality Constraints. Our approach is to construct an exact penalty function, and prove that the global minimal points of this exact penalty function are the primal constrained global minimal points. Thus we convert the problem of global constrained optimization into a problem of global unconstrained optimization.

Furthermore, the integral approach for finding a global minimization for a class of discontinuous functions is used and an implementable algorithm is given.

1. Introduction

In this paper, we will discuss the following problem: Find a global minimization:

(P_1) $\min f(x)$, s.t. $x\in S=\{x\in X:g_i(x)\leqslant 0,\ i\in I_0;\ h_j(x)=0,\ j\in J_0\}$, where I_0, J_0 are finite index sets; f, g, h_j, $i\in I_0$, $j\in J_0$, are continuous functions, $X\subset R^n$ is a bounded closed domain, and S is a nonempty set.

The integral method for finding a global minimization for a class of discontinuous functions in [1] will be used. So we begin by introducing this method and some related concepts.

Moreover, our approach is to construct an exact penalty function whose global minimal points are primal constrained global minimal points. Thus we convert the problem of global constrained optimization into a problem of global unconstrained optimization.

Definition 1. *The set $G\subset R^n$ is said to be a robust set if $\mathscr{I}(\operatorname{int} G)=\mathscr{I}G$, where* int *$G$ denotes the interior set of G, and* cl *G the closure of G. For example, the open set is a robust set; the closed set may not be a robust set; the set consisting of one point is a closed set, but not a robust set.*

Definition 2. *The function f is said to be a super robust function, if for any real c, the set $H_c^0=\{x:f(x)<c\}$ is a robust set.*

The super robust function has the following properities:

(1) A super semi-continuous function is a super robust function.

(2) If f is a super robust function, then so is af $(a>0)$.

(3) If f is a super robust function and g is a super semi-continuous function, then $f+g$ is a super robust function.

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(4) Let f be a super robust function and let the level set $\{H_c = x:\ f(x) \leqslant c\}$ be nonempty. If $\mu(H_c)=0$, then c is the global minimization of f, and H_c is the set of global minimal points.

(5) If D is an open set and G is a robust set, then $G \cap D$ is a robust set; if $G_i (i=1, \cdots, m)$ are robust sets, then $\bigcup_{i=1}^{m} G_i$ is a robust set.

Algorithm. Choose $c_0 > \min_{x\in X} f(x)$ and $H_{c_0} = \{x \in X:\ f(x) \leqslant c_0\}$ for $\mu(H_{c_0})>0$; otherwise c_0 is the global minimal value and H_{c_0} is the set of global minimal points. Let

$$c_1 = \frac{1}{\mu(H_{c_0})} \int_{H_c} f(x)\, d\mu,$$
$$H_{c_1} = \{x \in X : f(x) \leqslant c_1\},$$
$$\vdots$$
$$c_k = \frac{1}{\mu(H_{c_{k-1}})} \int_{H_{c_{k-1}}} f(x)\, d\mu,$$
$$H_{c_k} = \{x \in X:\ f(x) \leqslant c_k\},$$
$$\vdots$$

Then we obtain

$$H_{c_0} \supset H_{c_1} \supset \cdots \supset H_{c_k} \supset \cdots,$$
$$c_0 \geqslant c_1 \geqslant \cdots \geqslant c_k \geqslant \cdots,$$
$$H^* = \bigcap_{k=0}^{\infty} H_{c_k}, \quad c^* = \lim_k c_k.$$

Theorem. *If f is a super robust function and a lower semicontinuous function, and for some real a, $H_a = \{x \in X:\ f(x) \leqslant a\}$ is a nonempty set, then c^* is the global minimal values of f, and H^* is the set of global minimal points.*

For details we refer to [1].

2. Constrained Global Minimization and Exact Penalty Function

Since it is impossible to find on computer $x \in X$ such that $g_i(x) \leqslant 0$, $i \in I_0$ and $h_j(x)=0$, $J \in J_0$, we propose problem $(P_1)_{\varepsilon_1}$ to replace problem (P_1): $(P_1)_{\varepsilon_1}$: $\min f(x)$, s. t. $x \in S_{\varepsilon_1} = \mathscr{I}\{x \in X:\ g_i(x) < \varepsilon_1,\ i \in I_0;\ |h_j(x)| < \varepsilon_1,\ j \in J_0\}$, where $\varepsilon_1 > 0$ is a small positive number. Obviously, S_{ε_1} is a robust set, int $S_{\varepsilon_1} \supset S$, $(P_1)_{\varepsilon_1}$ is an approximation of (P_1), and $(P_1)_{\varepsilon_1} \xrightarrow[\varepsilon_1 \to 0]{} (P_1)$

The corresponding exact penalty function $(PC\varepsilon)_{\varepsilon_1}$ for $(P_1)_{\varepsilon_1}$ is

$$(PC\varepsilon)_{\varepsilon_1}:\ \min_{x\in X} F_{\varepsilon_1}(x, c, \varepsilon),\quad F_{\varepsilon_1}(x, c, \varepsilon) = f(x) + cP_{\varepsilon_1}(x, \varepsilon),$$

where

$$P_{\varepsilon_1}(x, \varepsilon) = \begin{cases} 0, & x \in S_{\varepsilon_1}; \\ \max\{g_i(x),\ i \in I_0;\ |h_j(x)|,\ j \in J_0;\ \varepsilon\}, & x \overline{\in} S_{\varepsilon_1}, \end{cases}$$

$\varepsilon > 0$ is a small positive number.

Lemma 2.1. *If g_i, $i \in I_0$, and h_j, $j \in J_0$, are continuous functions, then $P_{\varepsilon_1}(x, \varepsilon)$*