Journal of Computational Mathematics, Vol.10, No.1, 1992, 21–28.

A NEW KIND OF SCHEMES FOR THE OPERATOR EQUATIONS*

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Abstract

This paper provides a new kind of three-layer explicit schemes for solving the operator equation. It has good stability, and suits particularly the semidiscrete problems arising from solving multi-dimensional parabolic-type equations by the finite element method. The amount of its computation time is far less than that of any other algorithm of the finite element method and less than that of various economical schemes of the difference method. If the accuracy of the nonstandard finite element schemes (2.7) is not enough, it can be improved using extrapolations.

§1. Introduction

In numerical computation, for evolution equations the explicit schemes with good stability are more economical than the implicit schemes. In particular, when solving the evolution equation with the finite element method, we usually obtain a large system of ordinary differential equations about the time t. To discretie for time t, usually implicit schemes are used, but the amount of computation time is very large. In [1], the author has got a kind of explicit schemes for heat equations in the case of one and two dimensions under a special subdivision. In this paper, we remove the restriction of the number of dimensions and subdivision and consider the general operator equations. We get a kind of three-layer schemes with good stability. Of course, we are especially interested in the explicit forms of these schemes.

We consider discrete schemes for the operator equation

$$\bar{B}\frac{d\alpha(t)}{dt} + \bar{A}\alpha(t) = \bar{f}(t), \quad \alpha(0) = \alpha_0. \tag{1.1}$$

* Received September 2, 1988.

For the concepts, definitions and notations in this paper, please refer to [2] (Chapters 1, 5, 6). For the operator equation (1.1), the standard form of three-layer schemes is

$$\begin{aligned} &By_{\overset{\circ}{t}} + \tau^2 R y_{\bar{t}t} + Ay = \varphi(t), \\ &y(0) = y_0, y(\tau) = y_1, 0 < t = n\tau < t, \\ &n = 1, 2, ..., \quad n_0 - 1, \quad t = n_0\tau. \end{aligned} \tag{1.2}$$

The solution of (1.2) may be presented in sum form $y = \bar{y} + \tilde{y}$, where $\bar{y}$ satisfies the homogeneous equation

$$By_{\overset{\circ}{t}} + \tau^2 R y_{\bar{t}t} + Ay = 0, \quad y(0) = y_0, \quad y(\tau) = y_1, \tag{1.2a}$$

and $\tilde{y}$ satisfies the inhomogeneous equation with homogeneous initial condition

$$By_{\overset{\circ}{t}} + \tau^2 R y_{\bar{t}t} + Ay = \varphi(t), \quad 0 < t = n\tau < t_0, \quad y(0) = y(\tau) = 0. \tag{1.2b}$$

§2. A New Kind of Schemes for the Operator Equations

We consider a kind of three-layer schemes of the finite-dimensional operator equation and their stability. In the finite-dimensional space, a linear operator is equivalent to a matrix. So, we let $\bar{A}$, $\bar{B}$ be matrices of order N and α_0 be a given vector of dimension N.

In (1.1), we make the following approximation: let

$$\bar{B} = \bar{B}_0 + \bar{B}_1 + \bar{B}_2, \tag{2.1}$$

where the diagonal elements of $\bar{B}_1$ are zero. Assume

$$\bar{B}_0 \frac{d\alpha(t)}{dt} \approx \bar{B}_0 \alpha_{\overset{\circ}{t}}(t), \tag{2.2}$$

$$\bar{B}_1 \frac{d\alpha(t)}{dt} \approx \bar{B}_1 \alpha_{\bar{t}}(t), \tag{2.3}$$

$$\bar{B}_2 \frac{d\alpha(t)}{dt} \approx \bar{B}_2 \alpha_t(t), \tag{2.4}$$

$$\bar{A}\alpha(t) \approx \bar{A}\alpha(t) + \tau^2 \frac{D_{\bar{A}}}{2} \alpha_{\bar{t}t}(t), \tag{2.5}$$

$$D_{\bar{A}} = \begin{pmatrix} \bar{a}_{11} & & & 0 \\ & \bar{a}_{22} & & \\ & & \ddots & \\ 0 & & & \bar{a}_{NN} \end{pmatrix}. \tag{2.6}$$