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SPLINE COLLOCATION APPROXIMATION TO PERIODIC SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS*

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Abstract

A spline collocation method is proposed to approximate the periodic solu tion of nonlinear ordinary differential equations. It is proved that the cubic periodic spline collocation solution has the same error bound $O(h^4)$ and superconvergence of the derivative at collocation points as that of the interpolating spline function. Finally a numerical example is given to demonstrate the effectiveness of our algorithm.

§1. Introduction

The numerical approximation to the periodic solution of an autonomous ordinary differential equation system has been brought into consideration for more than a decade. Many numerical methods like the shooting method, Newton method, the linear multistep method etc. have been used in approximating the periodic solutions[1–5], but hardly any rigorous analysis of the convergence and error estimate of numerical solutions is given. In this paper a spline collocation method is introduced to approximate the periodic solution of ODEs. It is proved that the cubic periodic spline collocation solution (including a periodic orbit and its period) has the same error bound $O(h^4)$ and superconvergence of the derivative at collocation points as that of the interpolating spline function.

Consider an autonomous ordinary differential equation system

$$\frac{dx}{dt} = f(x). \tag{1.1}$$

Finding a T-periodic solution of (1.1) is equivalent to solving a non-trivial solution of the following boundary value problem[2].

$$\frac{dx}{dt} = Tf(x), \quad \frac{dT}{dt} = 0\ ; \tag{1.2}$$

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$$x(0) = x(1), \quad p(x) = 0, \tag{1.3}$$

where p is a functional of $C([0,1])$, which is known as a phase condition (Refer to [3] for further information). Here we choose

$$P(x(0)) = \begin{cases} x_k(0) - \alpha, & \text{if the } k\text{-th component of } x(0) \text{ is given}, \\ f_k(x(0)), & \text{if the } k\text{-th component of } x(t) \text{ takes its extremum at } t = 0. \end{cases} \tag{1.4}$$

Suppose $(x(t), T)$ is a solution of (1.2)-(1.3), $X(t)$ and $Y(t)$ are denoted respectively as the resolvents of

$$\frac{d\xi}{dt} = Tf'(x(t))\xi \tag{1.5}$$

and

$$\frac{d\xi}{dt} = -\frac{1}{3}Tf'(x(t))\xi. \tag{1.6}$$

If the matrix $J = \begin{pmatrix} I - X(1) & f(x(0)) \\ p'(x(0)) & 0 \end{pmatrix}$ is nonsingular, then the pair $(x(t), T)$ is a regular solution of (1.2)-(1.3)[3-4].

§2. The Cubic Spline Collocation Method

Let $\Delta = \{t_i\}_{i=1}^{N} (t_i = ih, h = \frac{1}{N})$ be a uniform partition of interval $[0,1]$, and $\{\phi_i(t)\}_{i=-1}^{N+1}$ be a cubic B-spline basis on mesh Δ. A pair $(x_h(t), T_h)$ is known as a cubic periodic spline collocation solution of (1.2)–(1.3), if $x_h(t) = \sum_{i=-1}^{N+1} c_i\phi_i(t)$ and T_h meets

$$\begin{cases} F_i(C_h) = x_h'(t_i) - T_h f(x_h(t_i)) = 0, & i = 1, \cdots, N, \\ F_{N+1}(C_h) = p(x_h(0)) = 0, \end{cases} \tag{2.1}$$

where $C_h = (c_1, \cdots, c_N, T_h)$, $c_{n+i} = c_i (i = -1, 0, 1), F_h = (F_1, \cdots, F_{N+1})$.

Lemma 2.1. *If $X(t)$ is a resolvent of (1.5), then*

$$f(x(0)) = X(1)\int_0^1 X(s)^{-1} f(x(s))ds. \tag{2.2}$$

Proof. Since $(x(t), T)$ is a solution of (1.2)–(1.3), $f(x(t))$ satisfies (1.5). So we have $f(x(t)) = X(t)f(x(0))$. Let $t = 1, f(x(0)) = f(x(1)) = X(1)f(x(0))$ .Therefore

$$X(1)\int_0^1 X(s)^{-1} f(x(s))ds = X(1)\int_0^1 f(x(0))ds = X(1)f(x(0)) = f(x(0)).$$