Journal of Computational Mathematics, Vol.11, No.1, 1993, 37–49.

THE SPECTRAL-DIFFERENCE METHOD FOR COMPRESSIBLE FLOW*

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Abstract

A spectral-difference scheme is proposed for semi-periodic compressible flow with strict estimation.

§1. Introduction

Tani[1] proved the existence of the local smooth solution of compressible viscous flow. In [2,3], a difference method and a spectral method were given. We consider the semi-periodic problem and use the spectral-difference method which has been successfully applied to fluid flow (see [4-7]).

Let n_1 and n_2 be positive integers. Let $x' = (x_1, \cdots, x_{n_1})^*, x'' = (x_{n_1+1}, \cdots, x_n)^*$ and $x = (x_1, \cdots, x_n)^*$. Let $\Omega = \Omega_1 \times \Omega_2$ where

$$\Omega_1 = \{x' | 0 < x_j < 1, 1 \le j \le n_1\}, \qquad \Omega_2 = \{x'' | 0 < x_j < 2\pi, n_1+1 \le j \le n\}.$$

We denote by Γ the boundary of Ω_1. The closures of Ω and Ω_l are denoted by $\overline{\Omega}$ and $\overline{\Omega_l}$. Let u be the velocity and $u = (u^{(1)}, \cdots, u^{(n)})^*$. p is the pressure. T is the absolute temperature. ρ is the density. f is the external force and $f = (f^{(1)}, \cdots, f^{(n)})^*$. $\nu(T, \rho) > 0$ is the viscosity, $\nu'(T, \rho)$ is the second viscosity and $\kappa(T, \rho) = \nu'(T, \rho) - \frac{2}{3}\nu(T, \rho)$. $\mu(T, \rho) > 0$ is the heat conduction coefficient. $S(T, \rho)$ is the entropy, $S_T = \frac{\partial S}{\partial T}$ and $S_\rho = \frac{\partial S}{\partial \rho}$. In order to avoid the instability of computation, we put $\varphi = \ln \rho$ as in [2]. For simplicity, we suppose that $p = R_0 \rho T$, R_0 being a positive constant. Then we have

$$\begin{cases} \dfrac{\partial u^{(l)}}{\partial t} + (u \cdot \nabla) u^{(l)} - e^{-\varphi} \dfrac{\partial}{\partial x_l}(\kappa \nabla \cdot u) - e^{-\varphi} \displaystyle\sum_{j=1}^{n} \dfrac{\partial}{\partial x_j}[\nu(\dfrac{\partial u^{(l)}}{\partial x_j} + \dfrac{\partial u^{(j)}}{\partial x_l})] \\ \quad + R_0 \dfrac{\partial T}{\partial x_l} + R_0 T \dfrac{\partial \varphi}{\partial x_l} = f^{(l)}, \quad l = 1, \cdots, n, \quad (x,t) \in \Omega \times (0, t_0], \\ \dfrac{\partial T}{\partial t} + (u \cdot \nabla) T - e^{-\varphi} T^{-1} S_T^{-1} (\nabla \cdot \mu \nabla) T - \dfrac{1}{2} \nu e^{-\varphi} T^{-1} S_T^{-1} \displaystyle\sum_{j,l=1}^{n} (\dfrac{\partial u^{(l)}}{\partial x_j} + \dfrac{\partial u^{(j)}}{\partial x_l})^2 \\ \quad - \kappa e^{-\varphi} T^{-1} S_T^{-1} (\nabla \cdot u)^2 - S_\varphi S_T^{-1} (\nabla \cdot u) = 0, \quad (x,t) \in \Omega \times (0, t_0], \\ \dfrac{\partial \varphi}{\partial t} + (u \cdot \nabla) \varphi + \nabla \cdot u = 0, \quad (x,t) \in \Omega \times (0, t_0]. \end{cases} \tag{1.1}$$

* Received September 4, 1990.

Suppose that all functions have the period 2π for the variable $x_j (n_1+1 \le j \le n)$ and that there exist positive constants $B_0, B_1, B_2, \nu_0, \nu_1, \kappa_1, p_0, \mu_0, \mu_1, S_0, S_1, S_2, \Phi_0$ and Φ_1 such that if

$$(T,\varphi) \in Q = \{(T,\varphi) | B_0 < T < B_1, |\varphi| < B_2\},$$

then $\left|\frac{\partial\eta}{\partial\varphi}\right|$ is bounded where $\eta = \nu, \kappa, \mu, S_T, S_\varphi$ and $q = T, \varphi$, and

$$\begin{aligned} &\nu_0 < \nu < \nu_1, \quad |\kappa| < \kappa_1, \quad \min(n\kappa + (n+1)\nu, \nu) > p_0, \\ &\mu_0 < \mu < \mu_1, \quad S_0 < S_T < S_1, \quad |S_\varphi| < S_2, \quad \Phi_0 < e^{-\varphi} < \Phi_1. \end{aligned} \tag{1.2}$$

§2. The Scheme and Error Estimation

Let J be a positive integer and $h = \frac{1}{J}$. The mesh domain is defined by

$$\Omega_{1,h} = \{x' | x_j = h, 2h, \cdots, 1-h, \text{ and } 1 \le j \le n_1\},$$

$$\overline{\Omega}_{1,h} = \Omega_{1,h} \cup \Gamma_h, \qquad \Omega_h = \Omega_{1,h} \times \Omega_2, \qquad \overline{\Omega}_h = \overline{\Omega}_{1,h} \times \overline{\Omega}_2,$$

where Γ_h is the boundary of $\Omega_{1,h}$, in the following form:

$$\Gamma_h = \bigcup_{j=1}^{n_1} \Gamma_j, \qquad \Gamma_j = \Gamma_{+j} \bigcup \Gamma_{-j},$$

$$\Gamma_{+j} = \{x' | x_j = 1 \text{ and } x_{j'} = h, 2h, \cdots, 1-h, \text{ for } j' \ne j\},$$

$$\Gamma_{-j} = \{x' | x_j = 0 \text{ and } x_{j'} = h, 2h, \cdots, 1-h, \text{ for } j' \ne j\}.$$

In addition, let $l'' = (l_{n_1+1}, \cdots, l_n), l''x'' = \sum_{j=n_1+1}^{n} l_j x_j$ and $|l''|_\infty = \max_{n_1+1 \le j \le n} |l_j|$, where $l_j (n_1+1 \le j \le n)$ is an integer. For any positive integer N, define

$$\overline{V}_N = \text{span}\{e^{il''x''} \mid |l''|_\infty \le N\}.$$

Let V_N be the subset of $\overline{V}_N$ involving all real valued functions. Let P_N be L^2-orthogonal projection from $L^2(\Omega_2)$ onto V_N. Let $\tau > 0$ be the mesh size of time t, and

$$\lambda = \max(2n_1\tau h^{-2}, n_2\tau N^2), \qquad Z_\tau = \left\{t = k\tau | k = 0, 1, \cdots, \left[\frac{t_0}{\tau}\right]\right\}.$$

Denote the value of the function η at point x and time $k\tau$ by $\eta(x,k)$, or simply by $\eta(k)$ or η. Assume that η, ξ, a, b are scalar functions and $w = (w^{(1)}, \cdots, w^{(n)})^*$. Define

$$\eta_{x_j}(x,k) = \frac{1}{h}[\eta(x+he_j,k) - \eta(x,k)], \qquad \eta_{\bar{x}_j}(x,k) = \eta_{x_j}(x-he_j,k),$$

$$\eta_{\hat{x}_j}(x,k) = \frac{1}{2}[\eta_{x_j}(x,k) + \eta_{\bar{x}_j}(x,k)], \qquad \eta_t(x,k) = \frac{1}{\tau}[\eta(x,k+1) - \eta(x,k)],$$

where e_j is the n-dimensional vector whose jth component equals 1 and the others are