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A GENERAL ALGORITHM AND SENSITIVITY ANALYSIS FOR VARIATIONAL INEQUALITIES*

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Abstract

The fixed point technique is used to prove the existence of a solution for a class of variational inequalities related with odd order boundary value problems and to suggest a general algorithm. We also make the sensitivity analysis for these variational inequalities and complementarity problems using the projection technique. Several special cases are discussed, which can be obtained from our results.

§1. Introduction

Variational inequality theory is a very useful and effective technique for studying a wide class of problems in a unified natural and general framework. This theory has been extended and generalized in several directions using new and powerful methods that have led to the solution of basic and fundamental problems thought to be inaccessible previously. Some of these developments have made mutually enriching contacts with other areas of mathematical and engineering sciences. We also remark that the theory so far developed upto now is only applicable to constrained boundary value problems of even order. On the other hand, little attention has been given to odd order boundary value problems. In recent years, the author has developed iterative type algorithms for a certain class of variational inequalities related with odd order boundary value problems having constrained conditions. We also study the qualitative behaviour of the solution of the variational inequalities when the given operator and the feasible convex set vary with a parameter. Such a study is known as sensitivity analysis, which is also important and meaningful. Sensitivity analysis provides useful information for designing, planning various equalibrium systems, predicting the future changes of the equilibria as a result of the changes in the governing systems. In addition, from a theoretical point of view, sensitivity properties of a mathematical programming problem can provide new insight concerning the problems being studied and can sometimes stimulate new ideas and techniques for solving them.

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Motivated and inspired by the recent research work going on in this area, we consider a new class of variational inequalities. Using the fixed point technique of Glowinski, Lions and Tremolieres [1], and Noor [2, 3], we prove the existence of a solution of these variational inequalities. This approach enables us to suggest and analyze a general algorithm for these variational inequalities. We also show that the variational inequality problem is equivalent to solving a fixed point problem using the projection method. This equivalence is used to analyze the sensitivity of the parametric variational inequality. This approach is due to Dafermos [4]. We also consider the sensitivity analysis for the general complementarity problems. Several special cases are also discussed.

In Section 2, we formulate the variational inequality problem and review some necessary basic results. The existence of the solution of the variational inequality problem is studied in Section 3 using the fixed point method along with a general algorithm. Sensitivity analysis is the subject of Section 4. The applications of the main results are considered in Section 5.

§2. Variational Inequality Formulation

Let H be a real Hilbert space with norm and inner product $\|.\|$ and $\langle\cdot,\cdot\rangle$ respectively. Let K be a nonempty closed convex set in H.

Given $T, g : H \to H$ continuous operators, consider the functional $I[v]$, defined by

$$I[v] = \frac{1}{2}\langle Tv, g(v)\rangle , \tag{2.1}$$

which is known as the general energy (cost) functional. Note that for $g = I$, the identity operator, then the functional $I[v]$, defined by (2.1) becomes

$$I_1[v]\frac{1}{2}\langle Tv,\ v\rangle ,$$

which is the classical energy functional.

If the operator T is linear, g-symmetric, that is

$$\langle T(u),\ g(v)\rangle = \langle g(u),\ T(v)\rangle, \text{ for all } u,\ v \in H,$$

and g-positive definite, then we can show that the minimum of $I[v]$, defined by (2.1) on the convex set K in H, is equivalent to finding $u \in H$ such that $g(u) \in K$ and

$$\langle Tu, g(v) - g(u)\rangle \leq 0, \text{ for all } g(v) \in K. \tag{2.2}$$

Inequality (2.2) is known as the general variational inequality, introduced and studied by Noor[5]. We remark that if $g = I$, the identity operator, then problem (2.2), is equivalent to finding $u \in K$ such that

$$\langle Tu,\ v - u\rangle \leq 0, \text{ for all } v \in K, \tag{2.3}$$

which is known as the variational inequality problem considered and studied by Lions and Stampacchia[6].