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ITERATIVE CORRECTIONS AND A POSTERIORI ERROR ESTIMATE FOR INTEGRAL EQUATIONS*

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Abstract

Starting from a well known operator identity we obtain a recurrence formula, i.e., an iterative correction scheme, for the integral equations with computable kernel. From this we can increase the order of convergence step by step, say, from 4th to 8th to 12th. What is more interesting in this scheme, besides its fast acceleration, is its weak requirement on the integral kernel: the regularity of the kernel will not be strengthened during the correction procedure.

§1. Operator Framework

Suppose that the linear operator equation

$$Lu = f \tag{1}$$

is approximated by another equation

$$L_0 u_0 = f, \tag{2}$$

where L_0 is an approximation of L in the sense of operator norm:

$$\varepsilon_0 \equiv \|L_0 - L\| \ll 1. \tag{3}$$

Thus, if L^{-1} exists, then L_0^{-1} exists:

$$\|L_0^{-1}\| \le (1 - \varepsilon_0\|L^{-1}\|)^{-1}\|L^{-1}\|. \tag{4}$$

For simplicity we will use the notation:

$$A_0 = L_0^{-1}(L_0 - L).$$

A well known identity

$$u - u_0 = A_0 u$$

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leads to a recurrence formula:

$$u - u_0 = A_0 u_0 + A_0^2 u = A_0 u_0 + A_0^2 u_0 + A_0^3 u = \cdots.$$

Setting

$$u_1 = A_0 u_0, \quad \ldots, \quad u_i = A_0^i u_0,$$

we obtain an iterative correction scheme:

$$u - \sum_{i=0}^{\gamma-1} u_i = A_0^\gamma u, \tag{5}$$

where the remainder is of high order:

$$\|A_0^\gamma u\| \le \|A_0\|^\gamma \|u\| \le O(\varepsilon_0^\gamma), \tag{6}$$

and u_i are nothing but the solutions of the same approximating equation (2) with different right hand sides:

$$L_0 u_i = (L_0 - L) u_{i-1}, \quad i = 1, 2, \cdots.$$

These u_i provide, as a by-product, an a posteriori error estimate.

Remark 1. The scheme (5) is nothing but a variant of formula (21) in [2]. If we set

$$L = I - K, \;\; L_0 = I - KP, \;\; \overline{u}^\gamma = \sum_{i=0}^{\gamma} u_i,$$

then formula (21) in [2] can be read as (5).

2. Iterated Galerkin Method

Let (1) be an integral equation of second kind:

$$Lu(s) \equiv u(s) - \int_0^1 K(s,t) u(t)\, dt = f(s) \tag{7}$$

and (2) the iterated Galerkin method:

$$L_0 u_0(s) \equiv u_0(s) - \int_0^1 K(s,t) P u(t)\, dt = f(s),$$

where P is the L_2-orthogonal projection onto a subspace with the standard approximation property:

$$\|u - Pu\| \le O(n^{-R}) \|\partial^R u\|,$$

where the lower order of derivatives on the right hand side are omitted.

Only one thing has to be done in applying the scheme (5), that is the estimate for (3). For this, we need the regularity condition on K:

$$\partial_s^R \partial_t^R K \in L_2 \tag{8}$$