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CALCULATIONS OF RIEMANN PROBLEMS FOR 2-D SCALAR CONSERVATION LAWS BY SECOND ORDER ACCURATE MmB SCHEME *1)

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Abstract

Numerical solutions of Riemann problems for 2-D scalar conservation law are given by a second order accurate MmB (locally Maximum-minimum Bounds preserving) scheme which is non-splitting. The numerical computations show that the scheme has high resolution and non-oscillatory properties. The results are completely in accordance with the theoretical solutions and all cases are distinguished efficiently.

1. Introduction

The initial value problem for 2-D scalar conservation law is

$$u_t + f(u)_x + g(u)_y = 0 \tag{1}$$

$$u(x,y,0) = u_0(x,y) \tag{2}$$

Define the region $\pi_T = [0,T) \times I\!R^2$, then the weak form and entropy condition of problem (1) (2) are

$$\int_{\pi_T} [u\phi_t + f(u)\phi_x + g(u)\phi_y] dxdydt + \int_{t=0} \phi u_0(x,y) dxdy = 0, \quad \forall \phi \in C_0^\infty(\pi_T) \tag{3}$$

and

$$\int_{\pi_T} sign(u-k)\{(u-k)\phi_t + (f(u)-f(k))\phi_x + (g(u)-g(k))\phi_y\} dxdydt \geq 0$$

$$\forall k \in I\!R, \quad \forall \phi \in C_0^\infty(\pi_T), \quad \phi \geq 0. \tag{4}$$

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We all know that in [1], existence and uniqueness of solution to problem (1)(2) have been obtained by using (3) and (4), and in [2], the two dimensional Riemann problem for (1)(2) has been solved analytically under the assumption

$$f''g''(f''/g'')' \neq 0.$$

There are many numerical methods for solving initial value problems of one dimensional conservation laws and practical problems. For all linear difference schemes, there exist two shortcomings that the solutions of first order accurate schemes are smoothed and the solutions of second order (or higher order) accurate schemes have oscillatory phenomena near discontinuities. In order to eliminate the shortcomings, several modified difference schemes (or nonlinear difference schemes) have been presented: Harten[3] proposed a second order accurate TVD difference scheme and gave a sufficient condition for the scheme to be TVD; Sweby[4] unified the work of van Leer[5], Roe[6] and Osher[7] and gave a class of second order accurate TVD schemes in the form of limiters; van Leer[8] constructed MUSCL scheme which generalized Godunov scheme to second order accuracy. In the previous papers, the TVD property of these schemes is valid only for one dimensional cases and has got great success to calculate problems of fluid dynamics. Unfortunately, in [9], it has proved that 2-D TVD scheme is at most of first order accuracy, although the splitting methods by using higher order one dimensional TVD schemes seem to work quite well for practical problems. In [10] [11] we presented a new MmB difference scheme which has high resolution, second order accurate and nonoscillatory properties, for initial value problems of 2-D scalar conservation laws.

In this paper, we briefly describe the theoretical solutions of two dimensional Riemann problems for scalar conservation law in section 2, and present the concept of MmB scheme and construct a class of second order accurate MmB schemes for 2-D conservation law in section 3; Finally, we show you all the configurations by numerical experiments.

2. Theoretical solutions

Consider problem (1)(2) and take initial data as follows,

$$u_0(x,y) = \begin{cases} u_1, & x>0,\ y>0 \\ u_2, & x<0,\ y>0 \\ u_3, & x<0,\ y<0 \\ u_4, & x>0,\ y<0 \end{cases}. \tag{5}$$

In [2], in order to construct the solutions of Riemann problem of (1)(5), the similarity transformation was used

$$\xi = x/t, \quad \eta = y/t$$

then $u(x,y,t) = \tilde{u}(\xi,\eta)$, (1) is exchanged as

$$(f'(\tilde{u}) - \xi)\tilde{u}_\xi + (g'(\tilde{u}) - \eta)\tilde{u}_\eta = 0.$$