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AN ARTIFICIAL BOUNDARY CONDITION FOR THE INCOMPRESSIBLE VISCOUS FLOWS IN A NO-SLIP CHANNEL*

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Abstract

The numerical simulation of the steady incompressible viscous flows in a no-slip channel is considered. A discrete artificial boundary condition on a given segmental artificial boundary is designed by the method of lines. Then the original problem is reduced to a boundary value problem of Navier-Stokes equations on a bounded domain. The numerical examples show that this artificial boundary condition is very effective and more accurate than Dirichlet and Neumann boundary conditions used in engineering literature.

1. Introduction

When computing the numerical solutions of viscous fluid flow problems in an unbounded domain, one often introduces artificial boundaries, and sets up an artificial bopundary condition on them; then the original problem is reduced to a problem on a bounded computational domain. In order to limit the computational cost, these boundaries must not be too far from the domain of interest. Therefore, the artificial boundary conditions must be good approximation to the "exact" boundary conditions (so that the solution of the problem in the bounded domain is equal to the solution in the original problem). Thus, the accuracy of the artificial boundary conditions and the computational cost are closely related. Designing artificial boundary conditions with high accuracy on a given artificial boundary has become an important and effective method for solving partial differential equations on an unbounded domain. In the last ten years, many authors have worked on this subject for various problems by different techniques. For details, refer to the works by Goldstein [1], Feng [2], Han [3,4,5], Hagstrom [6], Halpern [7] and Nataf [8] and the references there in.

The purpose of this paper is to design discrete artificial boundary conditions for the steady incompressible viscous flow in stream function vorticity formulation in the case when the domain is a no-slip channel. We use a direct method of lines [9] in the exterior domain and design discrete artificial boundary conditions at the segmental artificial

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boundaries. Then the original problem is reduced to a bounded computational domain. Furthermore, the numerical examples show that the artificial boundary condition given in this paper is very effective and more accurate than the Dirichlet and Neumann boundary conditions used in engineering literature.

2. Navier-Stokes Equations and Their Linearization

In this paper, we consider the numerical simulation of a steady incompressible viscous flow arounding a body (domain Ω_i) in a no-slipping channel defined by $\Re\times[0, L]$. Let u, v denote the components of the velocity in the x and y coordinate directions, and p denote the pressure. Then u, v and p satisfy the following Navier-Stokes (N-S) equations in domain $\Omega = \Re \times (0, L) \setminus \bar{\Omega}_i$:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = \nu \triangle u, \tag{2.1}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = \nu \triangle v, \tag{2.2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.3}$$

and boundary conditions

$$u|_{y=0,L} = v|_{y=0,L} = 0, \quad -\infty < x < +\infty, \tag{2.4}$$

$$u|_{\partial\Omega_i} = v|_{\partial\Omega_i} = 0 \tag{2.5}$$

$$u(x,y) \to u_\infty(y) = \alpha y(L-y), \quad v(x,y) \to v_\infty = 0, \qquad \text{when } x \to \pm\infty, \tag{2.6}$$

where $\nu > 0$ is the kinematic viscosity, and $\alpha > 0$ is a constant.

Introduce the stream function ψ and vorticity ω. Then

$$\frac{\partial \psi}{\partial y} = u, \quad \frac{\partial \psi}{\partial x} = -v, \tag{2.7}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{2.8}$$

Then the problem (2.1)–(2.6) is reduced to the following problem:

$$\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} - \nu \triangle \omega = 0, \quad \text{in } \Omega, \tag{2.9}$$

$$\triangle \psi + \omega = 0, \quad \text{in } \Omega, \tag{2.10}$$

$$\psi|_{y=0} = 0, \quad \psi|_{y=L} = \psi_L \equiv \int_0^L u_\infty(s)ds, \quad -\infty < x < +\infty, \tag{2.11}$$

$$\frac{\partial \psi}{\partial y}|_{y=0,L} = 0, \quad -\infty < x < \infty, \tag{2.12}$$