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GENERAL INTERPOLATION FORMULAS FOR SPACES OF DISCRETE FUNCTIONS WITH NONUNIFORM MESHES*1)

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Abstract

The unequal meshsteps are unavoidable in general for scientific and engineering computations especially in large scale computations. The analysis of difference schemes with nonuniform meshes is very rare even by use of fully heuristic methods. For the purpose of the systematic and theoretical study of the finite difference method with nonuniform meshes for the problems of partial differential equations, the general interpolation formulas for the spaces of discrete functions of one index with unequal meshsteps are established in the present work. These formulas give the connected relationships among the norms of various types, such as the sum of powers of discrete values, the discrete maximum modulo, the discrete Hölder and Lipschitz coefficients.

1. Introduction

The great number of problems for the large scale scientific and engineering computations concern the numerical solutions of various problems for the partial differential equations and systems in mathematical physics. The finite difference method is the most commonly used in these computations. So the theoretical and numerical studies of the finite difference schemes for the problems of the partial differential equations and systems naturally call people's great attentions.

The imbedding theorems and the interpolation formulas for the functions of Sobolev's spaces are very useful in the linear and nonlinear theory of the partial differential equations [1-4]. It is natural that the analogous extensions of the interpolation formulas for the discrete functional spaces must play the extremely important role in the study of the finite difference approximations to the problems of linear and nonlinear partial differential equations and systems. The discrete interpolation formulas and their consequences can be used in the study of the convergence and stability of the finite difference schemes for the various problems of linear and nonlinear systems of partial differential equations of different types. And they can also be used to construct the

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weak, generalized and classical, local and global solution for the problems of partial differential equations and systems. [5-11]

The finite difference schemes with unequal meshsteps for the problems of partial differential equations are much more complicated than the schemes with equal meshsteps. There are only very few simple results concerning this topic. Establishment of the general interpolation formulas for the spaces of discrete functions with unequal meshsteps obviously gives the possibility and strong apparatus for the systematic studies of the finite difference schemes with unequal meshsteps for the problems of partial differential equations.

The purpose of the present work is to establish a series of general interpolation formulas for the discrete functional spaces of discrete functions with equal and unequal meshsteps. These general interpolation formulas give the connected relationship among the discrete norms as the summations of powers, the maximum modulo and the Lipschitz and Hölder quotients for different discrete functional spaces. Also a series of consequences, derivations and applications for these interpolation formulas are justified. They are very commonly used in the further study for the finite difference approximations to the theory of partial differential equations.

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Let us divide the finite interval $[0,l]$ into the small segments by the grid points $\{x_j|j=0,1,\cdots,J\}$, where $0=x_0<x_1<\cdots<x_{J-1}<x_J=l$, J is an integer and $h_{j+\frac{1}{2}}=x_{j+1}-x_j>0(j=0,1,\cdots,J-1)$ are the equal and unequal meshsteps. The discrete function $u_h=\{u_j|j=0,1,\cdots,J\}$ is defined on the grid points $\{x_j|j=0,1,\cdots,J\}$ with unequal in general meshsteps $h=\{h_{j+\frac{1}{2}}|j=0,1,\cdots,J-1\}$. Let us denote $\triangle_+u_j=u_{j+1}-u_j$ or simplely $\triangle u_j=u_{j+1}-u_j(j=0,1,\cdots,J-1)$ and $\triangle_-u_j=u_j-u_{j-1}(j=0,1,\cdots,J)$.

Now let us introduce some notations of the difference quotients for the discrete function $u_h=\{u_j|j=0,1,\cdots,J\}$. As the discrete functions we take the notation for the difference quotient of first order

$$\delta u_h=\Big\{\delta u_{j+\frac{1}{2}}=\frac{u_{j+1}-u_j}{h_{j+\frac{1}{2}}}\Big|j=0,1,\cdots,J-1\Big\}, \tag{1}$$

which can be regarded as a discrete function defined on the grid points

$$\Big\{x^{(1)}_{j+\frac{1}{2}}=\frac{1}{2}(x_{j+1}+x_j)\Big|j=0,1,\cdots,J-1\Big\}.$$

of the interval $[x^{(1)}_{\frac{1}{2}},x^{(1)}_{J-\frac{1}{2}}]$ of length $x^{(1)}_{J-\frac{1}{2}}-x^{(1)}_{\frac{1}{2}}=l-\frac{1}{2}(h_{\frac{1}{2}}+h_{J-\frac{1}{2}})$ with the unequal in general meshsteps

$$\Big\{h^{(1)}_{j+\frac{1}{2}}=h_{j+\frac{1}{2}}\Big|j=0,1,\cdots,J-1\Big\}.$$