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L^∞ CONVERGENCE OF QUASI-CONFORMING FINITE ELEMENTS FOR THE BIHARMONIC EQUATION[*1)]

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Abstract

In this paper we consider the L^∞ convergence for quasi-conforming finite elements solving the boundary value problems of the biharmonic equation and give the nearly optimal order L^∞ estimates.

1. Introduction

The author has considered the L^∞ error estimates of conforming and nonconforming finite elements for the biharmonic equation. This paper will discuss the case of quasi-conforming finite elements.

Let Ω be a convex polygonal domain. For $p \in [1, \infty]$ and $m \geq 0$, let $W^{m,p}(\Omega)$ and $W_0^{m,p}(\Omega)$ be the usual Sobolev spaces, $\|\cdot\|_{m,p,\Omega}$ and $|\cdot|_{m,p,\Omega}$ be the Sobolev norm and semi-norm respectively. When $p = 2$, denote them by $H^m(\Omega)$, $H_0^m(\Omega)$, $\|\cdot\|_{m,\Omega}$ and $|\cdot|_{m,\Omega}$ respectively. Let $H^{-m}(\Omega)$ be the dual space of $H_0^m(\Omega)$ with norm $\|\cdot\|_{-m,\Omega}$. $\alpha = (\alpha_1, \alpha_2)$ is called a multi-index with $|\alpha| = \alpha_1 + \alpha_2$ if α_1 and α_2 are nonnegative integers. Define $0 = (0,0), e_1 = (1,0), e_2 = (0,1)$. For a multi-index α, let

$$D^\alpha = \frac{\partial^{|\alpha|}}{\partial x^{\alpha_1} \partial y^{\alpha_2}}$$

be the derivative operator.

Let M be the number of all multi-indexes α with $|\alpha| \leq m$. Define $L^{m,p}(\Omega) = (L^p(\Omega))^M$. For convenience, denote the components of $w \in L^{m,p}(\Omega)$ by $w^\alpha, |\alpha| \leq m$. Then $L^{m,p}(\Omega) = \{w | w = (w^\alpha), w^\alpha \in L^p(\Omega), |\alpha| \leq m\}$. For $w \in L^{m,p}(\Omega)$, define its norm $\|\cdot\|_{m,p,\Omega}$ and semi-norm $|\cdot|_{m,p,\Omega}$ as follows,

$$\|w\|_{m,p,\Omega} = (\sum_{|\alpha| \leq m} \int_\Omega |w^\alpha|^p dxdy)^{1/p}, \quad |w|_{m,p,\Omega} = (\sum_{|\alpha| = m} \int_\Omega |w^\alpha|^p dxdy)^{1/p}, \tag{1.1}$$

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when $p < \infty$, and

$$\|w\|_{m,\infty,\Omega} = \max_{|\alpha|\le m} \operatorname*{esssup}_{(x,y)\in\Omega} |w^\alpha(x,y)|, \quad |w|_{m,\infty,\Omega} = \max_{|\alpha|=m} \operatorname*{esssup}_{(x,y)\in\Omega} |w^\alpha(x,y)|, \tag{1.2}$$

when $p = \infty$. If $p = 2$, $\|\cdot\|_{m,p,\Omega}$ and $|\cdot|_{m,p,\Omega}$ can be written as $\|\cdot\|_{m,\Omega}$ and $|\cdot|_{m,\Omega}$ respectively.

Sobolev space $W^{m,p}(\Omega)$ or its subspace, by correspondence $u \in W^{m,p}(\Omega) \to (D^\alpha u) \in L^{m,p}(\Omega)$, is mapped to a subspace of $L^{m,p}(\Omega)$. Because the norm and semi-norm are invariant , it is also denoted by the usual notation.

For $h \in (0, h_0)$ with $h_0 \in (0,1)$, let T_h be a subdivisions of Ω by triangles or rectangles. Let $h_T = \operatorname{diam} T$ and ρ_T the largest of the diameters of all circles contained in T. Assume that there exists a positive constant η, independent of h, such that $\eta h \le \rho_T < h_T \le h$ for all $T \in \mathsf{T}_h$.

For $w \in L^2(\Omega)$ and $w|_T \in H^m(T)$ for all $T \in \mathsf{T}_h$, define

$$|w|_{m,h} = \left(\sum_{T\in\mathsf{T}_h} |w|^2_{m,T}\right)^{1/2}. \tag{1.3}$$

For $w \in L^\infty(\Omega)$ and $w|_T \in W^{m,\infty}(T)$ for all $T \in \mathsf{T}_h$, define

$$|w|_{m,\infty,h} = \max_{T\in\mathsf{T}_h} |w|_{m,\infty,T} \ . \tag{1.4}$$

The remains of the paper is arranged as follows. In section 2 we give the L^∞ estimates for 9-parameter quasi-conforming element for the biharmoic equation and its properties. In section 3 we present the proof of the L^∞ estimate for the element. In section 4 we consider the case of other quasi-conforming plate elements.

2. The 9-Parameter Quasi-Conforming Finite Element

The homogeneous Dirichlet boundary value problem of the biharmonic equation is the following,

$$\begin{cases} \triangle^2 u = f, & \text{in } \Omega \\ u|_{\partial\Omega} = \dfrac{\partial u}{\partial N}\Big|_{\partial\Omega} = 0 \end{cases} \tag{2.1}$$

where $N = (N_x, N_y)$ is the unit normal of $\partial\Omega$.

It is known that for $\forall f \in H^{-1}(\Omega)$, problem (2.1) has unique solution $u \in H_0^2(\Omega) \cap H^3(\Omega)$, such that

$$\|u\|_{3,\Omega} \le C\|f\|_{-1,\Omega}, \tag{2.2}$$

with C a positive constant.