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PREDICTOR-CORRECTOR ALGORITHM FOR CONVEX QUADRATIC PROGRAMMING WITH UPPER BOUNDS*

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Abstract

Predictor-corrector algorithm for linear programming, proposed by Mizuno et al.[1], becomes the best well known in the interior point methods. The purpose of this paper is to extend these results in two directions. First, we modify the algorithm in order to solve convex quadratic programming with upper bounds. Second, we replace the corrector step with an iteration of Monteiro and Adler's algorithm[2]. With these modifications, the duality gap is reduced by a constant factor after each corrector step for convex quadratic programming. It is shown that the new algorithm has a $O(\sqrt{n}L)$-iteration complexity.

1. Introduction

The predictor-corrector method for linear programming is a well known interior point method developed by Mizuno et al.[1], due to its quadratically convergent analysis. This kind of analysis usually contains two steps, i.e., predictor step and corrector step as one iteration. The corrector step is used only to ensure that the iterates stay close to the central path so that large step can be taken during the predictor step. The duality gap remains unchanged at corrector step for linear programming, but in case of convex quadratic programming, as shown later of this paper, this gap even increases. In this paper, we extend these results in order to solve convex quadratic programming with upper bounds. The predictor directions generated by our algorithm are similar to those generated by the algorithm presented in [1]. However, the corrector directions are replaced by the Monteiro and Adler's algorithm[2]. With these modifications, the duality gap is reduced by a constant factor after each corrector step. Therefore, we obtain a faster algorithm for convex quadratic programming.

The paper is organized as follows. In section 2, we outline the procedure of a predictor-corrector method. In section 3, we present convergence results for the algorithm. Final section contains further discussions.

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2. The Algorithm

We consider the following quadratic programming problem in standard form

$$
\text{(QP)} \qquad \begin{array}{ll} \min & c^T x + \frac{1}{2} x^T Q x \\ \text{s.t.} & \text{Ax} = \text{b}, \\ & x + z = d, \\ & x \geq 0, \quad z \geq 0, \end{array}
$$

where $c \in R^n, A \in R^{m\times n}$, $b \in R^m$, $d \in R^n$, and $Q \in R^{n\times n}$ are given, and Q is positive semi-definite, $x \in R^n$, $z \in R^n$, and the superscript T denotes the transpose. The standard logarithmic barrier interior point method is to incorporate the inequalities into a logarithmic barrier term and then append it to the objective function to obtain the following problem

$$
\text{(QP}_\mu\text{)} \qquad \begin{array}{ll} \min & c^T x + \frac{1}{2} x^T Q x - \mu \sum_{i=1}^n \ln x_i - \mu \sum_{i=1}^n \ln z_i \\ \text{s.t.} & \text{Ax} = \text{b}, \\ & x + z = d. \end{array}
$$

The first order optimality conditions for (QP) are

$$Ax - b = 0, \tag{1}$$
$$x + z - d = 0, \tag{2}$$
$$A^T y + s - w - Qx - c = 0, \tag{3}$$
$$XSe = 0, \tag{4}$$
$$ZWe = 0, \tag{5}$$
$$x, z, s, w \geq 0, \tag{6}$$

where X, S, Z and W are diagonal matrices with the elements x_i, s_i, z_i, and w_i respectively, y, w and s are dual variables and e denotes the n dimensional vector of all 1's Similarly, the first order optimality conditions for (QP$_\mu$) are

$$Ax - b = 0, \tag{7}$$
$$x + z - d = 0, \tag{8}$$
$$A^T y + s - w - Qx - c = 0, \tag{9}$$
$$XSe - \mu e = 0, \tag{10}$$
$$ZWe - \mu e = 0. \tag{11}$$

The primal-dual method, proposed by Monteiro and Adler[2], Carpenter [3], applies Newton's method directly to (7)-(11). Denote by F the set of all (x, z) and (y, s, w) that are feasible for the primal and dual, respectively. Denote by F^0 the set of all