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ALGEBRAIC-GEOMETRY FOUNDATION FOR CONSTRUCTING SMOOTH INTERPOLANTS ON CURVED SIDES ELEMENT*1)

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Abstract

The aim of this paper is to lay a algebraic geometry foundation for constructing smoothing interpolants on curved side element. Some interpolation theorems in polynomial space are given. The main results effectively for CAGD are presented.

1. Introduction

The algebraic geometry theory plays a very important role in many areas. For example, the fundamental theorem in the multivariate polynomial interpolation theory[8] is established by means of Bezout's theorem, the fundamental theoretical frame of multivariate spline functions is generated by means of compatible co-factor method[8] which is also closely related with Bezout's theorem[9].

The piecewise smooth function skill is used frequently in CAGD, FEC and scattered data fitting, etc. In using the skill, the smoothing interpolation scheme and its explicit representation on a given element are required. For a line grid partition of a given polygonal region, many results and effective skills in the application fields published by many mathematicians recently[1,2,4]. But few papers for the case of curved side element, which is also need to be considered in the application field, are found yet. In this case, the author will prove in section 3 that it is impossible to established Ženišek′s[10] type theorem on any curved sides element in any polynomial space. We consider the problem by using rational function. In fact, the results of this paper are sequel of the author's paper concentrating on the case of line grid partition element in which the generalized wedge function method for rational spline is introduced.

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2. Notations, Definitions and Preliminaries

Let Ω be a simply or multiply connected bounded region with piecewise algebraic curved boundary in the real plane $\mathbf{R^2}$. Denote by $\mathbf{P_k}$ the set of bivariate polynomials of total degree $\leq k$.

A real algebraic curve of order n in $\mathbf{R^2}$ is defined by the set

$$\{(x, y)|p(x, y) = 0\},$$

where $p(x, y) \in \mathbf{P_n}$ is a real polynomial of degree n ($n = \deg p(x, y)$). A real polynomial $p(x)$ is called to be irreducible in $\mathbf{R^2}$, if it can not be expressed a multiplication of any two real polynomials $g_1(x, y), g_2(x, y)$, $\deg g_i(x, y) > 0$. The divisor of algebraic curve F on G in the complex projective plane is defined by

$$F \circ G = \sum_p (m_p(F) \cdot m_p(G))p$$

where $m_p(F)$ and $m_p(G)$ are the multiplicity of F and G at p, respectively, and the symbolic summation is over all p, including neighbors.

The definition of polypol in R^2, curved sides element, is introduced by Wachspress[7]. Let curve C_m have n distinct irreducible components: $\Gamma_1, \Gamma_2, \cdots, \Gamma_n$. Let point v_1 in $\Gamma_n \circ \Gamma_1$, v_2 in $\Gamma_1 \circ \Gamma_2, \cdots v_n$ in $\Gamma_{n-1} \circ \Gamma_n$ be designated as vertices. Let $\bar{\Gamma_i}$ denote a given segment of curve Γ_i between vertices v_i and v_{i+1}(with $v_{n+1} = v_1$) such that the n segments define a simple closed planar figure. This figure is an algebraic element and is said to be *well-set (n-pol of degree m) polypol*[7] iff

(a) the vertices are all ordinary double-point of C_m,

(b) the (open) segments Γ_i contain only simple points of C_m,

(c) the polypol interior contains no point of C_m.

If a polypol is only satisfied (a) and (c), the polypol is called *ill-set polypol*.

The following two theorems are crucial results of the classical algebraic geometry.

Theorem 1. (Bezout's[9]) *The order of $F \circ G$ is $O(F \circ G) = \sum_p m_p(F) \circ m_p(G)$. If F and G have no common component in the complex projective plane, then $O(F \circ G) = t \cdot s$, where t and s are the order of F and G,respectively.*

For the case in the real plane, the following weak form of Bezout's theorem[8] is stated.

Bezout's Theorem (weak form). *If two real algebraic curves $C_1 : p_1(x, y) = 0$ and $C_2 : p_2(x, y) = 0$, of orders m resp. n, have $N > mn$ intersection points in the complex plane, then there is a real polynomial $Q(x, y)$ of degree $< min\{m, n\}$ such that*

$$p_1(x, y) = Q(x, y)R_1(x, y),$$

$$p_2(x, y) = Q(x, y)R_2(x, y),$$