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A MODIFIED BISECTION SIMPLEX METHOD FOR LINEAR PROGRAMMING*

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Abstract

In this paper, a modification of the bisection simplex method[7] is made for more general purpose use. Organized in an alternative simpler form, the modified version exploits information of the optimal value, as does the original bisection method, but no bracket on the optimal value is needed as part of input; in stead, it only requires provision of an estimate b_0 of the optimal value and an estimate of the error bound of b_0 (it is not sensitive to these values though) . Moreover, a new, ratio-test-free pivoting rule is proposed, significantly reducing computational cost at each iteration. Our numerical experiments show that the method is very promising, at least for solving linear programming problems of such sizes as those tested.

1. Introduction

The bisection method, proposed in an earlier paper by the author[7], exploits information of the optimal value to speed up the solution process. However, the method requires a bracket on the optimal value as part of its input, and its promisingly good performance depends on whether a suitable bracket is available; it may even fail to solve a problem if the initial interval provided does not contain the optimal value actually. In this paper, the method is modified for more general purpose use. Organized in an alternative simpler form, the new version no longer needs any bracket on the optimal value as part of input; in stead, it only requires an estimate b_0 of the optimal value and an estimate of the error bound of b_0, to which it is not sensitive though.

Nevertheless, it might be the new pivoting rule proposed that makes a more important improvement. The original rule (Rule 3.9 of [7]) of the bisection method may be regarded as a variant of Dantzig's classical rule, applied in an alternative administration; features of rules of this type are as follows:

(1) The incoming variable takes a feasible value.

(2) The outgoing variable takes the value of zero.

(3) All the feasible variables remain feasible after a basis change.

Although these classical conditions are widely accepted, and employed in different contexts, some authors such as Wolfe[13,14], Greenberg[4], Maros[5] and Belling-Seib[1]

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suggest relaxing condition (3); they reduce the amount of total infeasibility in stead. Rules of this type usually do require less iterations than classical methods. Unfortunately, they give a rise in computational cost per iteration. The new proposed rule, which is a ratio-test-free one, not only relaxes condition (3) but also gets rid of measuring infeasibility, consequently reducing computational effort at each iteration. Such type of rules have been very successful in other contexts[8,9,10,11]. Since numerical results of our tests show that the number of iterations required by the modified version is slightly less than that required by the original bisection method, total computational cost is reduced.

In Section 2, we propose the pivoting rule first, and then establish a procedure using the rule. In Section 3, we describe the modified algorithm in which the procedure is employed as its subalgorithm. Finally, in Section 4, we report our numerical results obtained, which are very encouraging although still preliminary.

2. The Ratio-Test-Free Rule

Consider linear programming problem in the standard form:

$$\max \ z = cx \tag{2.1a}$$

$$\text{s.t. } Ax = b \tag{2.1b}$$

$$x \geq 0, \tag{2.1c}$$

where $A \in R^{m \times n}$, $b \in R^m$, and c and x are row and column n-vectors, respectively.

In this section, an attempt is made for achieving feasibility under some *fixed* objective function value. For this purpose, the procedure, given in Section 3 of [7], is modified in an alternative simpler form in which a ratio-test-free pivoting rule is employed.

View $cx = z$ as a constraint, and take it as the 0-th constraint among others. Then, setting

$$A := \begin{pmatrix} c \\ \hline A \end{pmatrix}, \qquad b := \begin{pmatrix} z \\ \hline b \end{pmatrix}, \tag{2.2}$$

allows to denote the augmented constraint system by (2.1b) again. Thus now we have $A \in R^{(m+1)\times n}$ and $b \in R^{m+1}$ with $a_{0j} \equiv c_j$, $j = 1, \ldots, n$ and $b_0 \equiv z$, which may be referred to as *objective value parameter*. Assume that $Ax = b$ is consistent with $\operatorname{rank}(A) = k + 1 \leq m + 1 < n$.

Let $B \in R^{(m+1)\times(k+1)}$ be the basis of A with the basic index set

$$J_B = \{j_0, \ldots, j_k\}. \tag{2.3}$$

Introduce notation

$$\bar{J}_B = \{1, \ldots, n\} \backslash J_B \ . \tag{2.4}$$

The corresponding canonical form can then be represented by the tableau below:

$$B^+ A \mid B^+ b, \tag{2.5}$$

where B^+ is the Moore-Penrose inverse of B.