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THE DIRECT KINEMATIC SOLUTION OF THE PLANAR STEWART PLATFORM WITH COPLANAR GROUND POINTS*1)

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Abstract

A procedure of computing the position of the planar Stewart platfrom with coplanar ground points is presented avoiding the computation of Groebner basis by standard algorithm. The polynomial system resulted is triangularized. The number of arithmetic operations needed can be predisely counted.

1. Introduction

The problem for computing the position of a Stewart platfrom has been widely studied for various cases. In this paper we will consider the case for which the ground points are coplanar and the fixations of the legs on the platfrom are coplanar. It is simpler than the coplanar platform in [1] where the ground is not necessary a plane. Due to this simplicity the computation can be carried out directly avoiding the computation of Groebner basis by standard algorithm and then from it to deduce condition for 40 complex solutions is presented.

In section 2 we give the polynomial system we have chosen for the problem. How to transform the system into a simpler one with less unknowns is presented in section 3. The elimination process for solving the simpler system is presented in section 4. In section 5 we relate the result obtained with Groebner basis and characteritic set. The examples and remarks are given in sections 6 and 7 respectively.

The work reported here was done on a SUN SPARC station 2 using MAPLE V release 2. A program has been written in MAPLE functions.

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2. The Polynomial System for the Problem

The coordinates of base points B_i where the fixed legs are on the ground, are given by $(x_i, y_i, 0), i = 1, \cdots, 6$. Since the platform is planar, we denote the coordinates of points M_i of fixation of the legs on it by (p_i, q_i) with respect to any rectangular coordinate system given in the platform plane. Let its origin be M with coordinate (x, y, z) and the direction cosines of its axes MP and MQ be (u_1, u_2, u_3) and (v_1, v_2, v_3) respectively. Thus the coordinates of M_i can be expressed as

$$(p_i u_1 + q_i v_1 + x, \quad p_i u_2 + q_i v_2 + y, \quad p_i u_3 + q_i v_3 + z).$$

Let the length of $B_i M_i$ be l_i. We have six equations

$$f_i := (p_i u_1 + q_i v_1 + x - x_i)^2 + (p_i u_2 + q_i v_2 + y - y_i)^2 + (p_i u_3 + q_i v_3 + z)^2 - l_i^2 = 0 \quad i = 1, \cdots, 6.$$

Another three obvious equation are

$$\begin{aligned} f_\tau &:= u_1^2 + u_2^2 + u_3^2 - 1 = 0, \\ f_s &:= v_1^2 + v_2^2 + v_3^2 - 1 = 0, \\ f_9 &:= u_1 v_1 + u_2 v_2 + u_3 v_3 = 0. \end{aligned}$$

These 9 equations in 9 unknowns $u_1, u_2, u_3, v_1, v_2, v_3, x, y$ and z form the fundamental system describing the problem. For any j the total degree of f_j with respect to its unknowns is 2.

Note that this formulation the distances between $M_j's$ and those between $B_j's$ are not used explicitly. And we have not supposed that all $M_j's$ are distinct as well as $B_j's$. It might be more flexible. When some of M_j and/or B_j properly coincide, we get various corresponding special cases.

3. The Transformed System

Using f_7, f_8, f_9 and introducing

$$\begin{aligned} u &:= u_1 x + u_2 y + u_3 z, \\ v &:= v_1 x + v_2 y + v_3 z, \\ w &:= x^2 + y^2 + z^2 \end{aligned}$$

the first six equations can be written as

$$f_{10+i} := p_i x_i u_1 + p_i y_i u_2 - p_i u + q_i x_i v_1 + q_i y_i v_2 - q_i v + x_i x + y_i y - \frac{1}{2} w + m_i = 0$$

where

$$m_i := \frac{1}{2}(l_i^2 - x_i^2 - y_i^2 - p_i^2 - q_i^2).$$